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ABSTRACT

This publication discusses the difficulties of calculating reliable income estimates for particular groups of people and presents a methodology for overcoming those obstacles. Basically, the procedure involves setting up cumulative distributions of housing values, rental values, and family incomes for each census tract. Average housing values and rental values are then computed for each block and used to calculate average family income for the block. A number of adjustments may be made to offset disparities in rental values or home values and to reflect any income differences between renters and home owners. Following a brief introductory section, the author carefully outlines his statistical method, including the various adjustments that can be applied in different situations. The following section is a lengthy step-by-step example which uses actual 1970 census data for a Philadelphia census tract. The final section consists of a computer package for calculating family income distribution within a census tract. (JG)

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MANUAL ON PROCEDURE FOR USING CENSUS DATA TO ESTIMATE BLOCK INCOME

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MANUAL ON PROCEDURE FOR USING CENSUS DATA TO ESTIMATE BLOCK INCOME

Table of Contents

SECTION 1 . . . . . Introduction

SECTION 2 . . . . . Procedure for Using Block Housing Data to Estimate Block Income

A . . . . . Theoretical Description of Method

1 . . . . . Direct Method: Nonstochastic

2 . . . . . Percentile-to-Percentile Method: Nonstochastic

3 . . . . . Direct Method: Stochastic

4 . . . . . Percentile-to-Percentile Method: Stochastic

B . . . . . Procedure Used to Compute the Adjustment Factor for Block Income Estimation

C . . . . . Closing of the Open-Ended Classes of Housing, Rental Values and Income

D . . . . . Adjustment for Differences in Income of Renters and Owners

E . . . . . "Cookbook" Outline for Procedure to be Used

F . . . . . General Explanations for "Unusual Appearing Income Numbers"

G . . . . . Sources of Data

SECTION 3 . . . . . A Step-by-Step Example of the Procedure

SECTION 4 . . . . . Computer Package for Block Income Procedure

SECTION 1

INTRODUCTION

Income is considered to be an important measure of economic well-being, a determinant of socio-economic status and thus a factor important in many types of social science analyses. Distributional effects of governmental policies, voter analysis, demand for goods and services, and socio-economic effects of background are just a few examples of the types of analyses that use income estimates. For all of these, the analysis would be improved by an improvement in the estimates.

Thus, to have more accurate estimates of personal income has been an objective of social scientists. Numerous articles have been written on the use of and problems with individual income numbers and with mean numbers for large numbers of individuals. The transitory versus permanent income issue is the major problem of individual data. Income at any point of time is made up of two parts: permanent and transitory income.[1] Permanent income can be thought of as the income flow a family receives and expects to continue to receive based on earning activities of the family, their education, their occupation, and their wealth. Transitory income can be thought of as chance income which can be positive or negative. This can be based on illness, other short-term interruptions to employment, or overtime. For most research, permanent income is thus the desired measure because it is the income to which consumers adopt their behavior.[2] For groups, the transitory components tend to average out so that, in general, the current average

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[1] For further discussion, see Milton Friedman, A Theory of the Consumption Function, a study by the National Bureau of Economic Research (Princeton: Princeton University Press) 1957.

[2] Ibid, p. 221.

income of the group equals the permanent average income of the group. For individuals, the use of large group means, however, ignores relative income differences. Further, the characteristics of the groups selected may well result in the mean incomes being highly correlated with other variables under analysis.[1]

The use of mean income numbers for a small group is suggested here as one way of trying to deal with both sets of problems. The methodology presented is a way to use housing information and income distribution information to obtain mean incomes for residents by race of a Census block (a small group). The data used are for Philadelphia and are available from the 3rd and 4th counts of the 1970 Census. The rationale for this procedure, using a small group, is that, since housing represents a significant share of a family's expenditures, and since a family tends to remain in a residence over a long period, its expenditures on housing are likely to be related to its permanent income rather than to its transitory elements. Thus, the block income estimates from this procedure can be regarded as something close to permanent income estimates.

Housing on any block tends to be relatively homogeneous so that assigning one income number to all the families on the block may appropriately eliminate transitory elements among their incomes at any point in time.

Using housing data to obtain income estimates is preferable to using housing data alone as a surrogate for income for several reasons. First, housing consists of both owner-occupied and renter-occupied units and a method which combines them is required: combining the two housing values

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[1]See Jean Crockett, "Technical Note on Bias in Estimating Income and Expenditures," Consumption and Savings II, ed. by Friend and Jones (Philadelphia: Wharton School of Finance) 1960, 213-22; and Michael T. Hannen, "Problems of Aggregation" in Blalock, H., ed., Causal Models in the Social Sciences (Chicago: Aldine) 1971.

directly involves what is probably an unwarranted assumption of homogeneity between present discounted values of owned housing and rents across all blocks in the city. Second, and more important, budget studies[1] show that, in general, housing expenditures are not a constant share of permanent income as incomes increase. While housing expenditures are greater for higher income classes, they are generally a lower proportion of permanent income for higher income classes than lower income classes. The use of housing expenditures alone, then, would be likely to misrepresent or underestimate differences in income. Third, and tied to this last point, is the additional fact that the income-housing relationship is not perfect, so using housing alone would not be an accurate proxy for income.[2]

The procedure used is to set up cumulative distributions by race of housing values, rental values, and family incomes for each tract. Average housing values and rental values (again by race) are computed for each block. These values are then assigned the percentile that corresponds to their value in the appropriate cumulative housing distribution. Next, adjustments are made to take account of the imperfect correlation between rental values and income, or owner-occupied values and income. Basically, if the relationship is not perfect, an adjustment is made toward the tract mean income; otherwise, no such adjustment is made. Then, the tract income value corresponding to these percentiles are read out of the cumulative

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[1]See for example, Sherman J. Maisel and Louis Winnick, "Family Housing Expenditures: Elusive Laws and Intrusive Variances," Consumption and Savings I, ed. by Friend and Jones (Philadelphia: Wharton School of Finance) 1960, 371-3.

[2]This is based on correlations run between owner value and income and renter values and income by tracts for Philadelphia. The average correlation between tract income and housing is .21, the average correlation between tract contract rent and income is .28.

income distribution and assigned to the block's renters and owners. These income numbers are then adjusted by corresponding renter and owner adjustments. These adjusted income numbers are then weighted by the number of renter-occupied and owner-occupied units, respectively, to obtain one average income number for all families on this block.

For example, if the mean monthly contract rental of a block's 10 Negro families is $60, and within that tract, $60 is equivalent to the 40th percentile of contract rentals, according to the tract cumulative distribution of rental values, then the 40th percentile is assigned to the block's Negro renter units. Next, in this case assume that the relationship between rental values and income is close but not perfect. Thus, instead of assigning the 40th percentile to renter values, an adjustment is made for the imperfect relationship between income and rent, and the resulting percentile assigned is the 41st percentile (1% closer to the tract mean). Going to the cumulative percentage income distribution of Negroes for this tract, the 41st percentile in the income distribution may be equivalent to an income of $6,500. Then assume renters' incomes are below the tract average income and the adjustment factor is .95. Multiply $6,500 by .95 to get $6,175.

This procedure is then followed for owner-occupied units and an income is assigned similarly to the owners. Assume the Census provides an owner-occupied value for the block's 30 Negro families of $8,750, and that this is found to be equivalent to the 35th percentile within the tract's owner-occupied units for Negroes. Assume further, that the relationship between owner-occupied values and incomes is near perfect so that, after a small adjustment, the block Negro owner-occupied units are assigned to the 35.4th percentile. Looking at the income distribution, the 35.4th percentile is, let us say, equivalent to $5,700. Next, assume that owners' income

are above the tract average so the owner adjustment factor is 1.1. Multiply $5,700 by 1.1 to get $6,270. Finally, the number of owner-occupied units, 10, and the number of renter-occupied units, 30, are multiplied by the income elements (10 x $6,270 and 30 x $6,175); these products are added and divided by 40 (30 + 10). This yields a Negro block weighted average income of $6,198.75. The details of this procedure, the programming package used, and a description of the data follow.

Estimates for every block in the city of Philadelphia are available in a separate volume, Block Income Estimates, City of Philadelphia--1960 and 1970. 1960 block income numbers are included to permit analysis of change over the ten-year period. Housing data by race are not available from the 1960 Census and, therefore, income data by race are not estimated.

SECTION 2

PROCEDURE FOR USING BLOCK HOUSING DATA TO ESTIMATE BLOCK INCOME

For many analyses of urban problems, it would be useful to have income numbers for smaller units than a census tract. The argument and procedure outlined here is a suggested method for estimating these numbers. In particular, 1970 Census data on block mean housing values, block mean contract rental values, tract distribution of housing values, tract distribution of block contract rental values, and tract distribution of income values are used to obtain estimates of income for each block in Philadelphia. With some adjustments, the same type of estimates were made for 1960.

The accuracy of the procedure was tested by comparing the tract mean incomes derived from our estimates with the actual tract mean incomes available from the Census data. The correlation coefficient between these was .965.

In this Section, where the method is developed, housing, and income values are designated as relating to the family. When the method is applied to the actual data, block information will be a surrogate for the nonexistent family data. A further caveat is in order: The distinction between contract rental values and owner-occupied housing values will be generally ignored in the description of the method. The term housing value is used.

In Part A, the theoretical description of the method is presented. First, a direct method for estimating family income from the relationship between the tract distribution of housing values and the tract distribution of income is developed in Equations (1)-(11). The method assumes nonstochastic relationships between income and housing--i.e., a relationship with no random disturbances.

Second, a method for moving from the percentile distribution of tract housing values to the percentile distribution of tract income values, as a

way of obtaining estimates of family income, is shown in Equations (12)-(16). This "percentile-to-percentile" method is shown to be identical to the first method in the nonstochastic case. It is the preferred method since it does not require the justification of normality.

Third, the modifications involved in the development of the direct procedure for the stochastic case are shown in Equations (2')-(11').

Finally, the "percentile-to-percentile" method, adjusted with the modifications required for the stochastic assumption, will be developed.

In Section B, the procedure used to compute the adjustment factor under stochastic conditions is described.

In Section C, procedures used in closing the open-ended classes of housing, rental values, and income are presented.

In Section D, the adjustments for differences in owner and renter incomes are discussed.

In Section E, the steps to be used in estimating block income data from block housing data are detailed. The same procedure, is used for Negroes and non-Negroes for 1970. (1960 data are not available on this basis.)

In Section F, explanations for "Unusually Appearing" Income Numbers are presented.

In Section G, the data sources are described.

A. Theoretical Description of Method

1. Direct Method: Nonstochastic

Let y = family income
H_1 = owner-occupied housing value of family } = H_i
H_2 = contract rental outlay of family

The linear relationship between the housing variable and income is:

(1) $H_i = a_1 y + a_0$ (non-stochastic form)

or

$$y = \frac{1}{a_1} H_i - \frac{a_0}{a_1}$$

let

$$b_1 = \frac{1}{a_1}, \quad b_0 = -\frac{a_0}{a_1}$$

$$(2) \quad \therefore y = b_1 H_i + b_0$$

Assume that the distribution of housing for the families in tract, $f(H_i)$, is normal, with mean, μ_{H_i} and variance, $\sigma^2_{H_i}$. Then, the distribution of y, $f(y)$ will also be normal with mean μ_y and variance, σ^2_y. From (2), it follows that

$$(3) \quad \mu_y = b_1 \mu_{H_i} + b_0$$

$$(4) \quad \sigma^2_y = b_1^2 \sigma^2_{H_i}$$

Solving for b_1 and b_0, we get from (4)

$$(5) \quad b_1 = \frac{\sigma_y}{\sigma_{H_i}}$$

then, substituting into (3) the value of b_1 obtained in (5)

$$\mu_y = \frac{\sigma_y}{\sigma_{H_i}} \mu_{H_i} + b_0$$

$$(6) \quad b_0 = \mu_y - \frac{\sigma_y}{\sigma_{H_i}} \mu_{H_i}$$

Using the data from a census tract, we can estimate μ_y and σ_y from the income frequency distribution, and μ_{H_i} and σ_{H_i} from the frequency distribution of the housing variables. Denote these estimates by $\hat{\mu}_y$, $\hat{\sigma}_y$, $\hat{\mu}_{H_i}$, and $\hat{\sigma}_{H_i}$.

Then,

$$(7) \quad \hat{b}_1 = \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}}$$

and

$$(8) \quad \hat{b}_0 = \hat{\mu}_y - \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}} \hat{\mu}_{H_i}$$

Substituting into (2), we get an equation which enables us to estimate y from a block's housing value, denoted $\hat{y}(H_i)$.

$$(9) \quad \hat{y}(H_i) = \hat{b}_1 H_i + \hat{b}_0$$

Replacing $\hat{b}_1$ and $\hat{b}_0$ by their values in (7) and (8), we get

$$(10) \quad \hat{y}(H_i) = \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}} H_i + \hat{\mu}_y - \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}} \hat{\mu}_{H_i}$$

Rearranging terms, (10) can be written as

$$(11) \quad \hat{y}(H_i) = \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}} (H_i - \hat{\mu}_{H_i}) + \hat{\mu}_y$$

2. Percentile-to-Percentile Method: Nonstochastic

It can be shown that the following "percentile-to-percentile" procedure gives the same estimates as those obtained from (11).

Let P_{Hi} be the percentile position in the housing distribution of a family with a housing value H_i, and P_y be the percentile position in the income distribution of the tract with a family income y. Under the assumption that $f(H_i)$ is normal.

$$(12) \quad P_{H_i} = F\left[\frac{H_i - \mu_{Hi}}{\sigma_{H_i}}\right]$$

Where the function, F, is the cumulative density of a standardized (zero mean, unit variance) normal variable. Similarly, under the assumption that f(y) is normal,

$$(13) \quad \frac{y - \mu_y}{\sigma_y} = F^{-1}[P_y],$$

where the function, F^{-1}, is the inverse of F. Then,

$$(14) \quad y = \mu_y + \sigma_y \cdot F^{-1}[P_y]$$

The "percentile-to-percentile" method calls for equating a family's percentile position in the income distribution to the family's percentile position in the housing distribution. That is,

$$(15) \quad P_y = P_{H_i}$$

Equations (12), (14), and (15) together provide the steps of the "percentile-to-percentile" method:

a. Compute $\hat{\mu}_{H_i}$, $\hat{\sigma}_{H_i}$, $\hat{\mu}_y$, $\hat{\sigma}_y$ from the data on housing and income distributions of the tract.

b. For a given family housing value, H_i, compute

$$\frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}}$$

c. Obtain, using the standard normal curve table of areas, P_{H_i}, [Equation (13)].

d. Set P_y equal to P_{H_i}. [Equation (15)].

e. Find $\hat{y}(H_i)$ from P_y, using $\hat{\mu}_y$ and $\hat{\sigma}_y$ as estimates of the parameters of the distribution of y.

If

$$P_{H_i} = F\left[\frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}}\right], \quad \frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}} = F^{-1}(P_{H_i}).$$

Since $P_y = P_{H_i}$,

$$\hat{y}(H_i) = \hat{\mu}_y + \hat{\sigma}_y \cdot F^{-1}[P_{H_i}].$$

Then,

$$\hat{y}(H_i) = \hat{\mu}_y + \hat{\sigma}_y \cdot \frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}}.$$

Rearranging terms slightly,

$$(16) \quad \hat{y}(H_i) = \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}}(H_i - \hat{\mu}_{H_i}) + \hat{\mu}_y,$$

which is identical to Equation (11). Thus, the "percentile-to-percentile" method is equivalent to the first method described, where the housing-income relationship is nonstochastic. (The justification of the "percentile-to-percentile" method does not require the assumptions of normality of f(y) and linearity of the housing-income relationship.) It is easy to show, following the same steps as above, that the "percentile-to-percentile" method works if $f(H_i)$ is log normal and the housing-income relationship is linear-in-the-logs.

3. Direct Method: Stochastic

The modifications involve the introduction of disturbance terms in Equations (1) and (2), so that (2) now reads

$$(2') \quad y = b_1 H_i + b_0 + v,$$

giving rise to the following revision of Equations (3) through (11):

$$(3') \quad \mu_y = b_1 \mu_{H_i} + b_0$$

$$(4') \quad \sigma_y^2 = b_1^2 \sigma_{H_i}^2 + \sigma_v^2$$

$$(5') \quad b_1 = \frac{\sqrt{\sigma_y^2 - \sigma_v^2}}{\sigma_{H_i}}$$

$$(6') \quad b_0 = \mu_y - \frac{\sqrt{\sigma_y^2 - \sigma_v^2}}{\sigma_{H_i}} \mu_{H_i}$$

$$(7') \quad \hat{b}_1 = \frac{\sqrt{\hat{\sigma}_y^2 - \sigma_v^2}}{\hat{\sigma}_{H_i}}$$

$$(8') \quad \hat{b}_0 = \hat{\mu}_y - \frac{\sqrt{\hat{\sigma}_y^2 - \sigma_v^2}}{\hat{\sigma}_{H_i}} \hat{\mu}_{H_i}$$

$$(10') \quad \hat{y}(H_i) = \frac{\sqrt{\hat{\sigma}_y^2 - \sigma_{v_{H_i}}^2}}{\hat{\sigma}_{H_i}} + \hat{\mu}_y - \frac{\sqrt{\hat{\sigma}_y^2 - \sigma_v^2}}{\hat{\sigma}_{H_i}} \hat{\mu}_{H_i}$$

$$(11') \quad \hat{y}(H_i) = \frac{\sqrt{\hat{\sigma}_y^2 - \sigma_v^2}}{\hat{\sigma}_{H_i}} (H_i - \hat{\mu}_{H_i}) + \hat{\mu}_y$$

σ_v is unknown, but it can be expressed in terms of the correlation coefficient, r, associated with the income-housing relationship.

Since

$$r^2 = 1 - \frac{\sigma_v^2}{\sigma_y^2},$$

$$r\sigma_y = \sqrt{\sigma_y^2 - \sigma_v^2}$$

Therefore,

$$(11'') \quad \hat{y}(H_i) = \hat{r} \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}} (H_i - \hat{\mu}_{H_i}) + \hat{\mu}_y,$$

where $\hat{r}$ is an estimate of r.

4. Percentile-to-Percentile Method: Stochastic

Corresponding to (11'') is an "adjusted percentile-to-percentile" method: Proceed as before for steps (a), (b), and (c). Modify step (d) as follows:

(d') Find $w_{H_i} = F^{-1}(P_{H_i}) = \dfrac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}}$

(d'') Let $w_y = rw_{H_i}$

(d''') Then set $P_y = F(w_y) = F(rw_{H_i})$

Observe that since $r < 1$, P_y will be closer to .5 than P_{H_i}. When

$$r = 1,\ w_y = w_{H_i}$$

so

$$P_y = P_{H_i},$$

as in the nonstochastic case discussed above. When $r = 0$, then H_i provides no information about y, so it is appropriate that y be estimated by the mean income of the tract, which is equal to the median for the normal distribution. Then,

(e') Find $\hat{y}(H_i)$ from P_y as computed, using steps (d'), (d''), and (d''').

As before,

$$\frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}} = F^{-1}(P_{H_i}) = w_{H_i}$$

Now,

$$\hat{y}(H_i) = \hat{\mu}_y + \hat{\sigma}_y \cdot F^{-1}(P_y)$$

$$w_y = F^{-1}(P_y)$$

So,

$$\hat{y}(H_i) = \hat{\mu}_y \cdot \hat{\sigma}_y w_y$$

$$= \hat{\mu}_y + \hat{\sigma}_y \cdot r w_{H_i}$$

$$= \hat{\mu}_y + \hat{\sigma}_y \cdot r\left[\frac{H_i - \hat{\mu}_{H_i}}{\hat{\sigma}_{H_i}}\right]$$

Rearranging the terms,

$$\hat{y}(H_i) = r \cdot \frac{\hat{\sigma}_y}{\hat{\sigma}_{H_i}}[H_i - [\hat{\mu}_{H_i}] + \hat{\mu}_y]$$

This is identical to (11''), showing that the "adjusted percentile-to-percentile" method, in the stochastic case as well as in the nonstochastic case, gives the same result as the direct method. (The "adjusted percentile-to-percentile" method is to be preferred to the direct method for the same reasons indicated for the nonstochastic relationship.)

B. Procedure Used to Compute the Adjustment Factor for Block Income Estimation

1970 Census data which cross-tabulate income with classes of housing values and classes of rental values were used to find the relationship between income and housing. The information consists of seven income classes cross-tabulated by seven classes of housing values and eight income classes cross-tabulated by eight gross rent categories. Basically, each category is assigned a number representing its median--the value that divides the distribution into two equal groups.

The procedure used is as follows:

1. For rental and housing values, the midpoint of each closed class is used--while $350 is used for the $200 or more rental interval, $3,500 is used for housing units in the less than $5,000 interval and $60,000 is used for units in $50,000 or more interval. These are the values used by the Census in their calculations.

2. For income numbers another procedure is needed since the Census computes an actual mean rather than one based on grouped-censored data. Again, the midpoints are used for the closed classes, including the lowest class. For the upper open-ended class, a more rigorous procedure is used.

   There are two alternative procedures to use for the upper open-ended class, $25,000 plus. For the first, the choice of method depends on the tract income distribution. If a tract has observations in the $25,000 to $49,999 class but no observations the $50,000 plus class in the tract income distribution, the same method as that used for all closed classes is used. The midpoint of this class, $37,500 is used in the calculations. For those

tracts with observations in the $50,000 plus category in the tract income distribution, another method is followed. For each of these tracts a median is calculated for the classes $25,000 and above using a Pareto distribution. (A similar procedure and the reason for using a Pareto distribution is explained in greater detail in the discussion on closing open-ended classes of income.)

The alternative procedure and the one used here is to fit Pareto distributions to the separate income distribution for housing and rental occupants for total, Negroes, and non-Negroes. These functions are fitted to the individual income distributions, beginning with $10,000 and then solved to calculate a median for the class $25,000+. The disadvantage to this alternative is that many of these renters or owner distributions contain very few observations in the $10,000 and over categories so the fitted distributions may be poor. The steps of this procedure are:

a. The reverse cumulative distribution is used to fit a Pareto distribution to individual tract data. The parameters of the distribution, α and y_o, are calculated. The cumulative function is

$$P(y) = \left(\frac{y_o}{y}\right)^{\alpha}$$

for all y greater than y_o.

b. P(y) is set equal to P(y) up to the open-ended interval $25,000 plus, less one-half of the percentage of units in the open-ended interval. (NOTE: P(y) in the case of the Pareto distribution is reversed.)

c. A y* is calculated which corresponds to the indicated P(y) by the following formula:

$$P(y) = \left(\frac{y_o}{y^*}\right)^{\alpha}$$

then solving for y*

$$y^* = \frac{y_o}{P(y)}1/\alpha$$

3. An R is then derived for total, Negro, and non-Negro to measure the correlation between the value of owner-occupied units and income and between the rental values and income. Six R's are calculated for each tract whenever the Census makes the data available.

a. The data are directly available for total and Negro. In order to do the calculations for non-Negroes, the Negro distribution is subtracted from the total distribution. When the base is too small, the Census withholds the information so that in tracts with few Negroes the data are not available for Negroes.

b. If no data are available on Negroes, the R for the total, Negro and non-Negro, population is used.

c. When there are no tract data cross-classifying income by owner-occupied values or income by rental values, another method is used. After comparing alternative methods, we decided to use an average of the adjoining tracts in this situation.

(1) Calculation of a simple regression between R values of the rental-income and housing-income relationship for those tracts in which both are available showed the R^2 is the only .31. We regard this as too low to provide a reliable estimate.

(2) Computation of the mean R value and standard deviations of the rental-income and owned housing-income relations for all tracts leads us to conclude that the standard deviations are too large to comfortably use the mean for missing values.

	$\bar{x}$	σ
Owner-occupied	.21	.16
Renter-occupied	.28	.15

(3) Tract income values were plotted showing adjoining tracts to be similar with regard to income (see attached chart). Thus, we decided to use an average of the R's of adjoining tracts for the missing value.

d. Occasionally an R was negative. This was generally the result of having very few observations in the particular housing income relationship or having a few high income families with low housing values. In these cases, the r for the total population for that type of housing was substituted for the subgroup.

C. Closing of the Open-Ended Classes of Housing, Rental Values, and Income

The Census data used in calculating the income figures of each Census tract-block contained open-end classes for the uppermost categories; that is, they contain no limit to the highest classes. This was true for the housing, rental and income figures for 1960 and 1970. In order to be able to use the procedure we planned, which utilizes a cumulative distribution, it was necessary to either close these open-end classes,[1] or the case of income, fit a distribution to be able to calculate block income numbers in this category.

1. Rental and Housing Values: To close the upper open-ended classes of housing and rental values, we used the highest average block values. First, we found the highest average block value. Second

[1] In the extremes in a cumulative distribution, zero percent of the units receive zero percent of the income while 100 percent of the units receive 100 percent.

we assigned the 100% cumulative rental and housing value to these block values. That is, 100% of the housing owners were assumed to have housing values equal to or less than this highest value and 100% of the renters were assumed to pay rents equal to or less than the highest rental value.

2. Income: No block statistics were available so another procedure had to be followed for income.

   a. The first step was to fit the data to an appropriate distribution. The ones generally associated with income are the log-normal and Pareto distributions. The upper part of many income distributions have been found to have a Pareto distribution.[1] This distribution, as applied to income, asserts that the logarithm of the cumulative percentage of units with incomes above a certain level is a negatively sloped linear function of the logarithms of that value.[2] The Pareto distribution describes the upper tail of income. A test was run to see if the plotted data approximated a straight line. Using the cumulative distribution of the entire city of Philadelphia, the last several points of the reverse cumulative distribution were plotted on log-log paper. The points approximated a straight line so we felt safe in assuming a Pareto distribution. (See Graphs 1 and 2.)

   b. The parameters of the distribution had to be calculated in order to close the distribution.

      The Pareto distribution has a density function

$$p(y) = \alpha A y^{-(\alpha+1)}$$

      The parameter A can then be defined as $A = y_o^{\alpha}$ when the Pareto distribution defines only the upper tail of the distribution.

      The function then becomes

$$p(y) = \frac{\alpha}{y_o}\left(\frac{y_o}{y}\right)^{(\alpha+1)}$$

      where y_o is the minimum level included in the distribution. The distribution is said to hold for all values y above y_o.

      The cumulative function is then

$$P(y) = \left(\frac{y_o}{y}\right)^{\alpha} \quad \text{for all } y > y_o.$$

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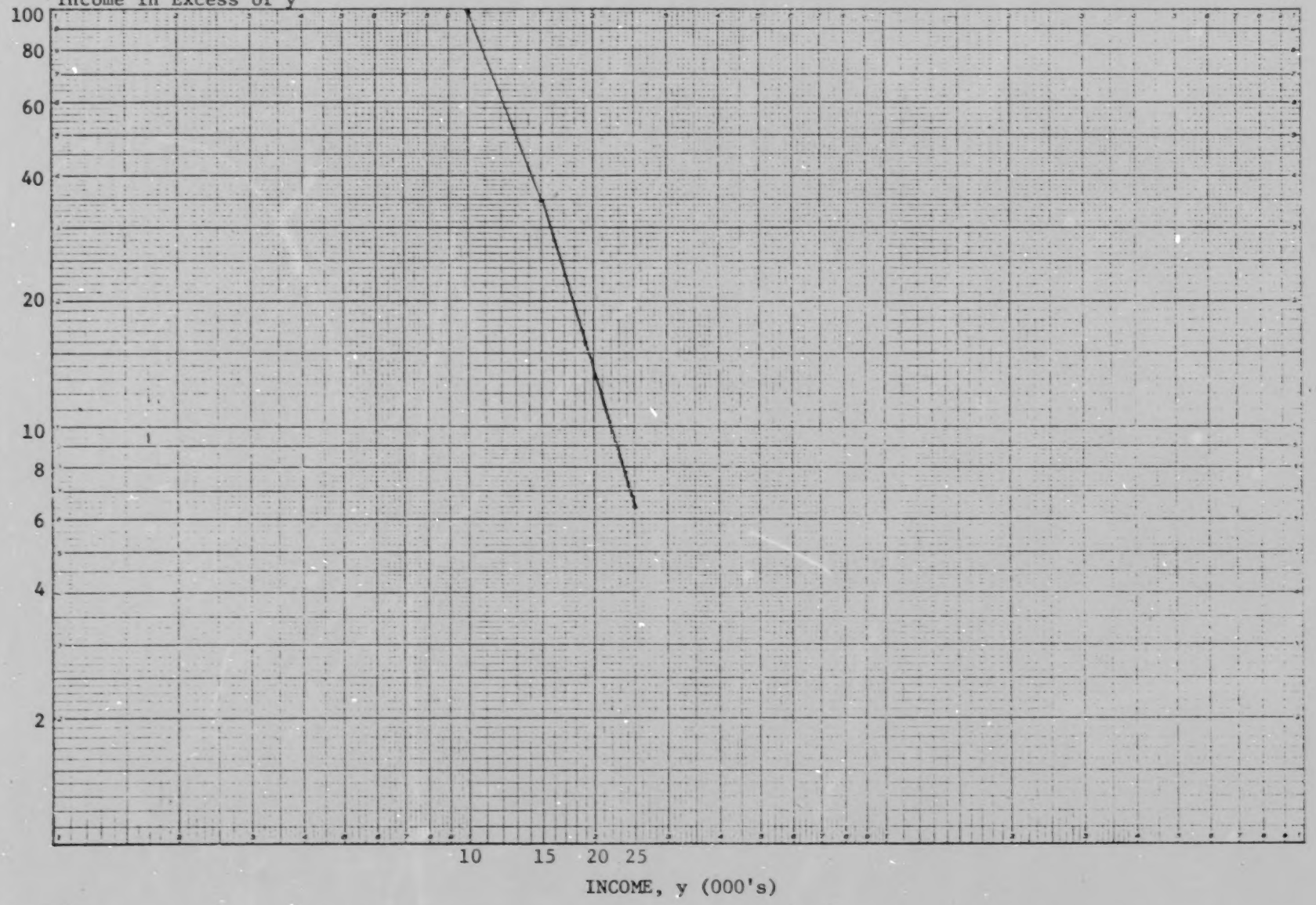
[1] See for example, L. R. Klein, An Introduction to Econometrics (Englewood Cliffs, N. J.: Prentice-Hall) 1962, Chapter 4, and C. E. Quensel as cited in J. Aitchison and J. A. C. Brown, The Lognormal Distribution (Cambridge: University Press) 1956, p. 108.

[2] Ibid., p. 150.

NO. 32.125. LOGARITHMIC 3 BY 2 3-INCH CYCLES. KEUFFEL & ESSER COMPANY, INC. NORWOOD, MASS. U.S.A.

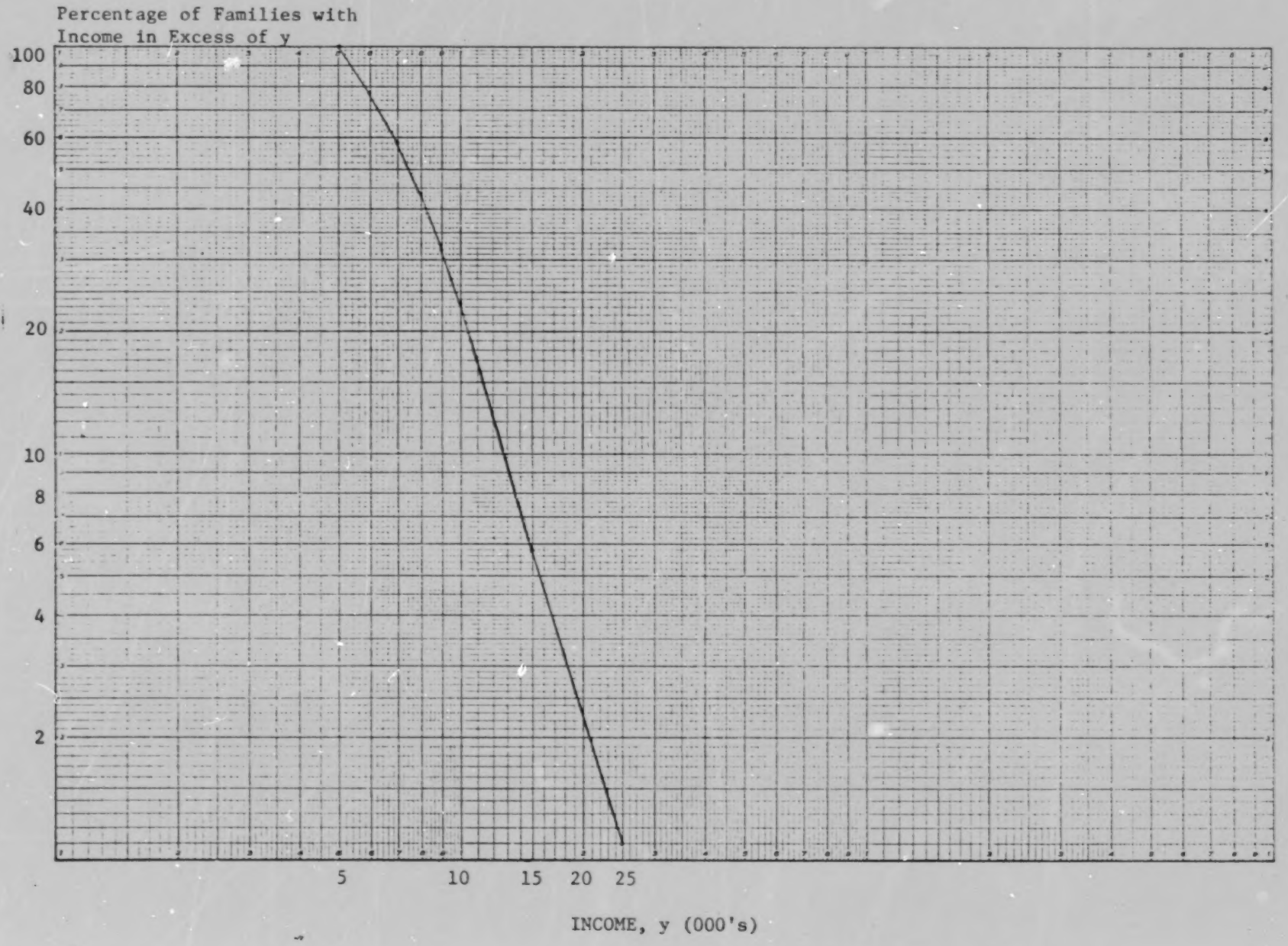
Graph 1
1970 CUMULATIVE INCOME DISTRIBUTION
CITY OF PHILADELPHIA, $10,000 PLUS



Graph 2

1960 CUMULATIVE INCOME DISTRIBUTION

CITY OF PHILADELPHIA, $5,000 PLUS



Once y_o is determined, the distribution depends on only one parameter, α. This is equivalent to the slope of the line plotted on the log-log paper. Basically then, we extended the line to close the distribution, to find the highest income level.

c. In order to find y_o, the minimum level of income, which best fit this distribution we varied the value of y_o. This yielded different estimates of $\hat{\alpha}$, and $\hat{P}(y)$ for each y above y_o, where $\hat{P}(y)$ is the estimated probability of observing a unit with an income level greater than y, such that

$$\hat{P}(y) = \left(\frac{y_o}{y}\right)^{\hat{\alpha}}$$

We then calculated the correlation coefficients between these $\hat{P}(y)$ and the P(y) from the actual income distribution for each $\hat{\alpha}$. The y_o which corresponds to the highest correlation coefficient was then used as the minimum value of the Pareto distribution and the $\hat{\alpha}$ associated with it as the slope of the distribution.

d. The fitted Pareto distribution itself is used to find a block's renter or owner income.

(1) For blocks with incomes in the highest category the y_o and α are fitted so only the P(y) is needed to solve for the income estimate.

(a) To get P(y) for each renter or owned block income over $50,000, the percentile associated with the renter or block value (PB) was noted.

(b) Since the Pareto distribution only holds for incomes above y_o, another adjustment was necessary. From the cumulative income distribution of the tract, the cumulative percentage associated with the y_o being used in the Pareto distribution, P_{yo} was found. Then, using this information, P(y) can be computed by the following formula:

$$P(y) = 1 - \frac{PB - P_{yo}}{(1 - P_{yo})}$$

(2) The equation

$$y^* = \frac{y_o}{[P(y)]^{1/\alpha}}$$

was then solved to get the renter's or owner's block income directly.

D. Adjustment for Differences in Income of Renters and Owners

It would be desirable to have separate income distributions for owners and renters so that the "percentile-to-percentile" method could correctly assign the corresponding owner or renter income to the owned housing value percentile and contract rental percentile.

While cross-classifications of income with owned values and gross rental values are available, we considered them less desirable to use than the overall tract income distributions and housing distributions for several reasons:

1. The data used in the cross-classification are from the 20% sample on housing. In order to weight it up to 100%, the Census uses a special weighting procedure for housing units which differs from the weighting procedure for persons or families, the procedure generally used for income.

2. The mean income values computed using the two income distributions from housing cross-classifications together differed significantly in most cases from the means computed by the Census. (See Table next page for illustrations.)

3. The income value we are interested in is family income, not housing unit income. For other purposes, however, researchers might find it useful to use this cross-classified data to create separate income distributions for renters and owners.

While we did not wish to use these separate income distributions directly they were the only data source available to adjust for expected differences in owners and renters income. (In general we expected higher owner incomes and lower renter incomes.[1])

Basically, we assume that while the renter-income and owner-income distributions do not measure the income concept we are interested in, they do adequately reflect differences between owners and renters, and so are of use in adjusting the income numbers.

Using these cross-classifications one can compute three income distributions: one for renters, one for owners, and one for renters and owners combined. These can then be used to compute various types of adjustment factors.

1. One alternative considered was to compute the mean income of renters, the mean income of owners, and the mean income of renters and owners together. Two ratios can then be computed:

---

[1] As incomes increase, the liklihood of owning increases. For example, Maisel and Winnick, op. cit., p. 395, in 1960 cited Survey of Consumer Expenditure evidence that the home-ownership rate rose from 35% for families with income between $2,000 and $3,000 to 82% for families with incomes over $10,000.

COMPARISON OF SAMPLE OF TRACT MEAN INCOMES FROM INCOME DISTRIBUTION AND FROM RENTER AND OWNER INCOME CROSS CLASSIFICATIONS,

City of Philadelphia
1970 Mean Income Estimates

Source	Tract Income Distribution	Owner and Renter Income Cross-Classifications Combined	Owner-Income Cross-Classification	Renter-Income Cross-Classification
Total				
Tracts				
Tract 1	$ 9,957	$ 9,308	$11,861	$ 7,119
Tract 2	7,029	3,485	8,705	2,970
Tract 3	13,167	3,310	15,571	8,273
Tract 4	22,183	10,037	--	10,037
Tract 10	20,912	19,316	44,550	13,701
Negro				
Tract 1	$10,900	--	--	--
Tract 2	3,525	$ 2,957	--	$ 2,957
Tract 3	3,500	10,771	--	10,771
Tract 4	18,200	7,061	--	7,061
Tract 10	7,407	6,566	--	6,566
Non-Negro				
Tract 1	$ 9,806	--	--	--
Tract 2	9,577	$ 2,973	--	$ 2,973
Tract 3	13,338	8,202	--	8,202
Tract 4	22,219	10,192	--	10,192
Tract 10	21,944	14,266	--	14,266

Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census 4th Count Population and Housing Summary Tapes, Philadelphia County.

(a) mean renters income/mean combined income and;

(b) mean owners income/mean combined income.

These factors could then be used as adjustments. The problem with this approach is that it may too heavily weight a few high observations and result in adjustments that are too great over most of the income range.

2. A second alternative was to compute the median income from three income distributions, then compute the two ratios by dividing renters median income by the combined median income and owners median income by the combined median income. This eliminates the heavy weight on a few high observations but assumes homegeniety of the difference between the two distributions throughout the distributions.

3. A third alternative, and the one used in this manual, is to compute, for each of the three income distributions, the income at the 25th, 50th (median), and 75th percentiles. Then compute six adjustment factors:

Let Y_{ir} = renter's income at the ith percentile.

Let Y_{io} = owner's income at ith percentile.

Let Y_{iro} = all housing units income at ith percentile.

Then the adjustment factors are:

(a) $\frac{Y_{25r}}{Y_{25ro}}$

(b) $\frac{Y_{25o}}{Y_{25ro}}$

(c) $\frac{Y_{50r}}{Y_{50ro}}$

(d) $\frac{Y_{50o}}{Y_{50ro}}$

(e) $\frac{Y_{75o}}{Y_{75ro}}$

(f) $\frac{Y_{75o}}{Y_{75ro}}$

The appropriate adjustment factor or computed adjustment factor is then used.

The first and second are used if the percentile of the block's renter or owner income number is 25 or below; the fifth and sixth are used if the percentile of the block's renter or owner income number is 75 or above. If the adjusted percentile falls between 25 and 75 the adjustment factors are interpolated.

This is the procedure used here. While it still requires certain homogeniety assumptions, it permits the cumulative distributions to differ.

4. A fourth alternative was to form the adjustment factors for all of the blocks within a tract. Basically,

$$\frac{Y_{ir}}{Y_{iro}}$$

and

$$\frac{Y_{io}}{Y_{iro}}$$

would be computed for all observed i's.

This procedure would be cumbersome and involve assumption of too close identity between the distributions.

E. "Cookbook" Outline for Procedure to be Used

1. Form the cumulative distribution of data for each tract of owner-occupied housing values, contract rental values, and family income. Then, convert these cumulative distributions into relative distributions (percentiles).

2. For tracts with observations in the $50,000 plus category, fit a Pareto distribution to the income distribution.

3. For each block, calculate the average owner-occupied housing values and contract rental values.

4. For each block find the percentile equivalents in the tract distribution of mean owner-occupied house value and mean contract rental value.

5. For each of these two percentiles, find the corresponding normal deviate values:

   a. Go into the body of the normal curve area table with the percentiles.

   b. Read out the normal deviate arguments.

(Because the normal curve is symmetric, the standard normal curve table of areas covers only the upper half of the curve. If a percentile value is greater than one-half, the procedures of a. and b. are straight-forward. If a percentile value is less than one-half, it cannot be found directly in the body of the table. Instead, find the normal deviate argument for 100 minus the percentile value and assign a negative sign to the argument.)

6. Compute the correlation coefficient, r, for the owner-occupied-income and renter-occupied-income relationships. Use Census procedure for assigning values to classes where possible. For open-ended income classes compute the median assuming a Pareto distribution.

7. Multiply the arguments by the appropriate r to get the adjusted arguments.

8. Find the percentiles corresponding to the adjusted arguments from the body of the normal curve table of areas.

9. With the percentile values just obtained, find the corresponding income values from the cumulative income distribution. If value falls above $50,000 use the Pareto distribution fitted to this tract to calculate income value.

10. Calculate homeowner and renter adjustments using the cross-classified information on rental value and income and owned values and income.

    a. Form three income distributions: Renters from renter cross-classification, owners from owner cross-classification, and renters and owners combined from these two distributions. Use the same income categories for all three.

    b. For each of these three income distributions calculate the income corresponding to the 25th, 50th, and 75th percentiles.

    c. Form six adjustment factors by dividing the renters and owners income at each of these percentiles by the income of the combined distribution at the corresponding percentiles.

11. Obtain the mean adjusted incomes. Multiply the renter and owner incomes by the appropriate adjustment factor. If renter income (or owner income) is calculated from a percentile 25 or below, use the renter adjustment factor (or owner adjustment factor) calculated at the 25th percentile. If renter (or owner) income is calculated from the percentile 75 or above, use the corresponding factor calculated at the 75th percentile. If the income percentile equivalent is between 25 and 75, interpolate between the corresponding adjustment factors; use the 25th and 50th when the percentile is greater than 25 and less than 50, and the 50th and 75th when the percentile is greater than 50 and less than 75.

12. Average the two income values obtained for each block, using as weights the number of units of owner-occupied housing and the number of units of contract rental housing.

F. General Explanations for "Unusual Appearing" Income Numbers

Several of the estimated income numbers may appear unusual. Included in this category are blocks where similar housing values lead to very different income numbers both across blocks and intrablock by racial subcategory, or where very different housing values lead to similar income numbers. Five examples are presented below with suggested explanations.

1. Rental values are significantly different but income numbers are very similar.

   Possible Explanations:

   (a) The correlation (r) between rent and income may be low so that there is a sizeable adjustment toward the mean.

   (b) The income distribution is heavily concentrated in any one category, for example, $10,000-$12,500, so that any observations that fall within this category have incomes assigned within the category.

2. The income figure assigned to the total category is not between the figures assigned the two racial subgroups.

   Possible Explanation:

   The income figures are based on a weighted average of incomes assigned to renters and owners. The data for a block may include one type of housing information for all three categories but information on the other type housing for only one subgroup and total. In this case the income figures of one subgroup and total will be an average of renter and owner income but the other subgroup will be the income of renters or owners only. For example, the data for a Block may include rental information for all three categories but owner-occupied information for total and non-Negro only so that the Negro income figure is based only on renters. The owner-occupied values are the same for both total and non-Negro but the renter value is higher for total than non-Negro. Thus the income value assigned to total may be higher than that assigned to non-Negroes and Negroes.

3. Similar housing figures for Negroes and non-Negroes have very different income numbers assigned to them.

   Possible Explanation:

   Negro and non-Negro income distributions frequently are quite different. For example, a Negro who spends $73 on rent may have an income of $3,187 while a non-Negro paying a similar rent has a higher income. (Their expenditures on housing as a percentage of income differ.)

4. Income appears low in relation to average housing value.

   Possible Explanation:

   The average income is based on a weighted average of renters and owners estimated incomes. In a block with few owners and many renters the income number is much closer to the renters income number. If renters income differs from the owners income, the average may appear too low or high. The type of Block that might fit this description is a Block with several small apartment houses or renter-occupied units and a few large single homes.

5. Owner and renter values identical for Negro subgroup and total group but income figures differ.

   Possible Explanation:

   Income, housing and rental distributions for Negroes and non-Negroes generally differ. Total is composed of both Negro and non-Negro so it generally differs from either one (it is a composite of both). The income value may differ (1) because the particular rental or housing value is assigned a different percentage. If this percentage is the same in both cases, it may differ (2) because the income distributions differ and the same percentage (for example, 42%) has a different income value associated with it.

G. Sources of Data

1. The sources of data for the 1970 block income figures are the Third Count Summary Tape and the Fourth Count Summary Tape of Population and Housing for Philadelphia based on the 1970 Census.

2. The third count tape is based on the complete count (i.e., the 100% sample). It contains the aggregate contract rent and count of the renter-occupied units by Census blocks and the aggregate value of owner-occupied units and count of these units by blocks. These data are also available by race. This information is used to compute average block rental and average block value of owner-occupied units. A count of renter-occupied and owner-occupied

units by eight categories of values from the 100% sample is used at the tract level to compute the cumulative distribution of these rental and housing values for the total population.[1] This count is not broken down by race.

3. The fourth count housing tape is based on a sample count. The items used in these computations are all based on the 20% sample. The data on monthly contract rent by 15 categories are available for total and Negro breakdowns. They are used to compute the cumulative distributions of rental and housing values for Negro and non-Negro (or white). The latter is computed by subtracting the count of values for Negroes from the total count by values.

4. The cross tabulations of income classes by monthly gross rents (eight categories) and income classes by owner-occupied values (seven categories) are used to get the R's--the adjustment statistics for each tract. This is also from the fourth count housing tape.[2] (NOTE: The data contains income for housing units not families so differs from the owned tract income distribution.)

5. The fourth count population tape contains data on family income by 15 income categories. They are based on the 20% sample, and are tabulated by Total, White, Negro, and Spanish-American. Three cumulative distributions are calculated: Total, Negro, and non-Negro (which is found by subtracting Negro from total).[3]

---

[1]This is used rather than the fourth count because it is based on the 100% sample, whereas the fourth count is based on the 20% sample.

[2]The tract family income distribution and income distribution based on the housing-income cross-classifications generally differ. The income measure we are interested in is family income, not income of housing units. Housing units may be for "a single family, one person living alone, two or more unrelated persons who share living arrangements." Both the income distribution and the housing-income distributions are based on a 20% sample. The Census uses ratio estimation procedures to weight the data and make it equal 100 percent. The procedures differ for housing units and families or persons. Persons are based on household type groups--age, sex, race, and head of household groups. Housing units are based on renter or owner groups and household type groups.

For more information see "Weighting the 1970 Census Samples," Small-Area Data Notes, Vol. 7, No. 1, January 1972 (U. S. Department of Commerce: Washington, D. C.).

[3]Ibid.

SECTION 3

A STEP-BY-STEP EXAMPLE OF THE PROCEDURE

To aid in using the procedure outline for going from block housing to block income data, an example using one tract in Philadelphia is worked, in detail, in this section. The step numbers match the "Cookbook" outline in Section 2E.

STEP 1 - Forming the Cumulative Distribution of Family Income, Owner-Occupied Housing Values and Contract Rental Values for the Tract and Converting to Cumulative Percentage Distributions.

Income Distribution. Tract 14's family income distribution taken from the 1970 4th Count Census of Population is converted into a cumulative distribution. Total and Black are directly available from the Census. Non-Black is obtained by subtracting the Black count from the Total count. The income intervals are from the Census. The count is converted into the cumulative count by adding each successive count to the sum of the earlier counts. In order to obtain the cumulative percentage, the cumulative count in each interval is divided by the total number of observations.

Example: In Table I, the total and Negro count of families by income interval are from the Census data. For the non-Negro count, the Negro count of 62 is subtracted from the total count of 62 to obtain 0 for the non-Negro count for the $0 - $999 interval. For the $2,000 - $2,999 interval, 138, the Negro count, is subtracted from 151, the total count, to get a non-Negro count of 13. This is done for all income intervals. Next, to compute the cumulative count for the total population, 62 is added to 58 to get 120, 151 is added to 120 to get 271, and so forth until 4, the count of the highest category is added to 1,021, the cumulative count up to that category to obtain 1,025, the total number of observations for the income distribution of Tract 14. The Negro and non-Negro cumulative count are

TABLE I

1970, PHILADELPHIA, TRACT 14: TRACT INCOME DISTRIBUTION
COUNT, CUMULATIVE COUNT, AND CUMULATIVE PERCENTAGE
BY TOTAL, NEGRO, AND NON-NEGRO

Income Intervals (in Dollars)	Total			Negro			Non-Negro		
	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage
$ 0 - 999	62	62	6.0	62	62	6.2	0	0	0.0
1,000 - 1,999	58	120	11.7	58	120	12.0	0	0	0.0
2,000 - 2,999	151	271	26.4	138	258	25.9	13	13	46.4
3,000 - 3,999	122	393	38.3	122	380	38.1	0	13	46.4
4,000 - 4,999	129	522	50.9	129	509	51.1	0	13	46.4
5,000 - 5,999	89	611	59.6	89	598	60.0	0	13	46.4
6,000 - 6,999	98	709	69.2	98	696	69.8	0	13	46.4
7,000 - 7,999	75	784	76.5	75	771	77.3	0	13	46.4
8,000 - 8,999	80	864	84.3	73	845	84.8	6	19	67.9
9,000 - 9,999	56	920	89.8	74	901	90.4	0	19	67.9
10,000 - 11,999	29	949	92.6	29	930	93.3	0	19	67.9
12,000 - 14,999	46	995	97.1	46	976	97.9	0	19	67.9
15,000 - 24,999	26	1021	99.6	21	997	100.0	5	24	85.7
25,000 - 49,999	0	1021	99.6	0	997	100.0	0	24	85.7
50,000 +	4	1025	100.0	0	997	100.0	4	28	100.0

Data Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Population Summary Tape, Philadelphia county.

TABLE II

1970, PHILADELPHIA, TRACT 14: DISTRIBUTION BY VALUE CLASSES OF OWNER-OCCUPIED HOUSING, TOTAL

Owner-Occupied Housing Value Intervals (in Dollars)	Count	Cumulative Count	Cumulative Percentage
$ 0 - $ 4,999	99	99	25.9
$ 5,000 - $ 9,999	242	341	89.3
$10,000 - $14,999	32	373	97.6
$15,000 - $19,999	8	381	99.7
$20,000 - $24,999	0	381	99.7
$25,000 - $34,999	0	381	99.7
$35,000 - $49,999	1	382	100.0
$50,000 +	0	382	100.0

Data Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 3rd Count Summary Tape, Philadelphia county.

TABLE III

1970, PHILADELPHIA TRACT 14: DISTRIBUTION BY VALUE CLASSES OF OWNER-OCCUPIED HOUSING BY NEGRO AND NON-NEGRO

Owner-Occupied Housing Value Intervals (in Dollars)	Negro			Non-Negro		
	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage
$ 0 - $ 4,999	96	96	27.0	3	3	14.3
$ 5,000 - $ 7,499	124	220	61.8	0	3	14.3
$ 7,500 - $ 9,999	89	309	86.8	11	14	66.7
$10,000 - $12,499	30	339	95.2	0	14	66.7
$12,499 - $14,999	3	342	96.1	0	14	66.7
$15,000 - $17,499	0	342	96.1	0	14	66.7
$17,500 - $19,999	14	356	100.0	7	21	100.0
$20,000 - $24,999	0	356	100.0	0	21	100.0
$25,000 - $34,999	0	356	100.0	0	21	100.0
$35,000 - $49,999	0	356	100.0	0	21	100.0
$50,000 +	0	356	100.0	0	21	100.0

Data Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census 4th Count Housing Summary Tape, Philadelphia county.

TABLE IV

1970, PHILADELPHIA, TRACT 14: DISTRIBUTION BY VALUE CLASSES OF RENTAL HOUSING, TOTAL

Contract Rental Intervals (in Dollars)	Count	Cumulative Count	Cumulative Percentage
$ 0 - $ 39	219	219	12.7
$ 40 - $ 59	972	1191	69.2
$ 60 - $ 79	465	1656	96.2
$ 80 - $ 99	55	1711	99.4
$100 - $119	5	1716	99.7
$120 - $149	3	1719	99.9
$150 - $199	0	1719	99.9
$200 +	2	1721	100.0

Data Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 3rd Count Summary Tape, Philadelphia county.

TABLE V

1970, PHILADELPHIA, TRACT 14: DISTRIBUTION BY VALUE CLASSES OF OWNER-OCCUPIED HOUSING BY NEGRO AND NON-NEGRO

Contract Rental Intervals (in Dollars)	Negro			Non-Negro		
	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage
$ 0 - $ 29	22	22	1.3	0	0	0.0
$ 30 - $ 39	116	138	8.2	6	6	17.6
$ 40 - $ 49	578	716	42.4	6	12	35.3
$ 50 - $ 59	501	1217	72.1	5	17	50.0
$ 60 - $ 69	315	1532	90.7	0	17	50.0
$ 70 - $ 79	118	1650	97.7	17	34	100.0
$ 80 - $ 89	28	1678	99.3	0	34	100.0
$ 90 - $ 99	5	1683	99.6	0	34	100.0
$100 - $119	6	1689	100.0	0	34	100.0
$120 - $149	0	1689	100.0	0	34	100.0
$150 - $199	0	1689	100.0	0	34	100.0
$200 - $249	0	1689	100.0	0	34	100.0
$250 - $299	0	1689	100.0	0	34	100.0
$300 +	0	1689	100.0	0	34	100.0

Data Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Housing Summary Tape, Philadelphia county.

computed similarly. Following these steps the cumulative percentage is calculated. For the total population, each cumulative count is divided by the total number of income observations, 1,025; thus, 62 divided by 1,025 is 6.0%. This process is continued until the last count 1,025 is divided by 1,025 to get 100%. For Negroes, 62 is divided by 997, the Negro total count, to get 6.2%. The same procedure is followed for non-Negroes.

Distribution of Owner-Occupied Housing Values. The total count of owner-occupied units by eight value classes is available from the third count tape of the 1970 Census, which is based on the 100% sample. Racial breakdowns are not available in this sample, however, so the fourth count tape based on the 20% sample is used for Negro and non-Negro. This contains more categories of owner-occupied values. In all cases, the actual count of units by value class is used to calculate the cumulative count and cumulative percentage distribution of owner-occupied housing values.

Example: Table II presents the count, cumulative count, and cumulative percentage for Tract 14 by the eight value classes for the total population. Table III presents the count, cumulative count, and the cumulative distribution for Negroes and non-Negroes by eleven value classes for Tract 14. The combined Negro and non-Negro count of owner-occupied units and numbers in each category as shown in Table III differ somewhat from the total count in Table II since these subgroups are based on a 20% sample and then weighted by a special census procedure for housing units, while the data on Table II are from the 100% sample. The non-Negro count is found by subtracting the Negro count from the 20% sample from the total count of the same 20% sample.

Distribution of Rental Values. The distribution for contract rental units is calculated in a way exactly like the owner-occupied housing values. For the total population of renters, the 100% sample is available from the

third count tape but for Negro and non-Negro (i.e., Total--Negro), only the fourth count based on the 20% sample is available.

Example: Table IV presents the count, cumulative count and cumulative percentage for Tract 14 by the 8 contract rental classes for the total population. Table V presents these counts for Negroes and non-Negroes by 14 value classes for Tract 14. Again, it is important to remember that the differences between Table IV and V occur because the Total is from the 100% sample while the racial subgroups are from the 20% sample.

STEP 2 - Fitting a Pareto Distribution to the Income Distribution for Tracts with Incomes above $50,000

In order to use the "percentile-to-percentile" method, it is necessary to assign income numbers to blocks that fall in the $50,000 plus category. To do this it is necessary to assume a certain distribution. The Pareto distribution was assumed after plotting an income distribution on double logarithmic graph paper to see if it approximated a straight line. The testing was done by plotting on the vertical axis the percentage of units with income above a certain income against, on the horizontal axis, the corresponding incomes. For the city of Philadelphia it approximated a straight line. (See Graph 1) For a Pareto distribution this line can then be extended to find the income associated with any percentage of units in the upper tail. The Pareto distribution has a cumulative function $P(y) = \left(\frac{y_o}{y}\right)^{\alpha}$ for all $y \geq y_o$. y_o is the minimum level included in the distribution, the beginning point for the straight line. This is so, since the distribution is generally only assumed to hold for the upper tail of the observation, because it does not graduate low levels of income satisfactorily.[1] The Pareto distribution assumes the probability of an observation

[1]See L. Klein, An Introduction to Econometrics (Englewood Cliffs, N. J.: Prentice Hall), 1962, p. 151.

TABLE VI

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION
RECALCULATED FOR PARETO DISTRIBUTION, y_o = 6,000

Observation	Income	Percent Total	Percent in Logs Total	(Log y_o - Log y)	Percent[1] Non-Negro	Percent in Logs[1] Non-Negro
1	6,000	1.000	0.0	0.0	1.00	0.0
2	7,000	.763	-0.117	-0.07	1.00	0.0
3	8,000	.582	-0.235	-0.12	1.00	0.0
4	9,000	.389	-0.410	-0.18	.60	-0.222
5	10,000	.254	-0.596	-0.22	.60	-0.222
6	12,000	.184	-0.736	-0.30	.60	-0.222
7	15,000	.073	-1.14	-0.40	.60	-0.222
8	25,000	.010	-2.01	-0.60	.267	-0.574
9	50,000	.010	-2.01	-0.92	.267	-0.574

[1]NOTE: There are no observations between 6,000 and 8,000 and between 9,000 and 15,000.

The resulting equation for total is: Percent in logs (total) = 2.541 (log y_o - log y)
(15.78)
R^2 =.93

The figure in parenthesis is the t statistic. The R^2 is adjusted for degrees of freedom.

TABLE VII

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION RECALCULATED FOR PARETO DISTRIBUTION, y_o = 7,000

Observation	Percent in Logs Total	Percent in Logs Non-Negro	(Log y_o - Log y)
1	0.0	0.0	0.0
2	- .118	0.0	-0.06
3	- .293	- .222	-0.11
4	- .478	- .222	-0.15
5	- .619	- .222	-0.23
6	-1.02	- .222	-0.33
7	-1.90	- .574	-0.55
8	-1.90	- .574	-0.85

The resulting equation for total is:

$$\text{Percent in logs (total)} = \underset{(13.17)}{2.636} (\log y_o - \log y) \qquad R^2 = .91$$

TABLE VIII

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION
RECALCULATED FOR PARETO DISTRIBUTION, y_o = 8,000

Observation	Percent in Logs Total	Percent in Logs Non-Negro	(Log y_o - Log y)
1	0.0	0.0	0.0
2	-0.175	- .222	-0.05
3	-0.361	- .222	-0.10
4	-0.501	- .222	-0.18
5	-0.905	- .222	-0.27
6	-0.780	- .574	-0.49
7	-1.780	- .574	-0.80

The equation fitted to this data for total is:

$$\text{Percent in logs (total)} = \underset{(10.76)}{2.688}(\log y_o - \log y) \qquad R^2 = .88$$

TABLE IX

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION RECALCULATED FOR PARETO DISTRIBUTION, y_o = 9,000

Observation	Percent in Logs Total	Percent in Logs Non-Negro	(Log y_o - Log y)
1	0.0	0.0	0.0
2	-0.185	0.0	-0.05
3	-0.326	0.0	-0.12
4	-0.729	0.0	-0.22
5	-1.606	-0.352	-0.44
6	-1.606	-0.352	-0.74

The resulting equation is:

$$\text{Percent in logs (total)} = \underset{(8.96)}{2.589}\ (\log y_o - \log y) \qquad R^2 = .86$$

TABLE X

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION RECALCULATED FOR PARETO DISTRIBUTION, y_o = 10,000

Observation	Percent in Logs Total	Percent in Logs Non-Negro	(Log y_o - Log y)
1	0.0	0.0	0.0
2	-0.140	0.0	-0.08
3	-0.544	0.0	-0.18
4	-1.420	-0.352	-0.40
5	-1.420	-0.352	-0.70

The resulting equation for total is:

$$\text{Log of Percent in logs total} = \underset{(7.36)}{2.431}\ (\text{Log } y_o - \text{Log } y) \qquad R^2 = .84$$

being below y_o is zero, so the sum of probabilities from y_o to closing must be 1.

For this tract, looking at Table I, we tried y_o's of \$6,000, \$7,000, \$8,000, \$9,000, and \$10,000. No Negroes are in the open-ended category--so the income distribution ends or is closed at \$50,000. For non-Negroes the same y_o's are tried as for the Total population: \$6,000, \$7,000, \$8,000, \$9,000, and \$10,000 (approximately the city-wide mean income).

The procedure for fitting a Pareto distribution to the tract income data is to run a linear regression of the logs suppressing the constant term.

Example:

1. $P(y) = \left(\frac{y_o}{y}\right)^{\alpha}$ is the cumulative distribution of the Pareto function where P(y) is the probability that an observation will have an income greater than or equal to income y. So, for Tract 14, Table I, gives the count of income units by income categories. The cumulative distribution is recalculated since the Pareto distribution only fits the top tail of the distribution. To do this, the count from y_o on is summed and the cumulative distribution calculated in reverse since the Pareto cumulative distribution is cumulated by incomes in excess of each successive income level. This is done for y_o's of \$6,000, \$7,000, \$8,000, \$9,000, and \$10,000 for Total and non-Negro in Tables VI-X. Table VI gives all the data; the others only show the changed data.
2. Next, logs are taken: $\log P(y) = \alpha(\log y_o - \log y)$, and α estimated by running a linear regression with log P(y) as the dependent variable and $(\log y_o - \log y)$ as the

independent variable suppressing the constant term. This is done for all 5 initial incomes y_o. For y_o = 6,000, subtract the log of each income from the log of 6,000, 3.78, to use as the independent variable. The logs of the cumulative distribution are the dependent variable. Run a linear regression in the logs suppressing the constant term.

3. Next, select the Pareto distribution that best fits the actual data. This is done by selecting the y_o that corresponds to the highest correlation coefficient, r^2. The r^2 measures the relationship between the predicted probabilities that a family will have an income above a certain level according to the distribution and the observed probabilities of the sample. The highest r^2 indicates that the predicted probabilities best match the observed probabilities. For Tract 14, comparing these five results, the best fit occurs using y_o = 6,000, where α = 2.541 and r^2 = .93.

4. Using the slope estimated from the best equation and the initial income level y_o, associated with it, it is possible to find the income equivalent of any percentile in the $50,000 plus range, using the formula:

$$P(y) = \left(\frac{y_o}{y^*}\right)^{\alpha}.$$

Then solve for y*, the desired income level.

$$y^* = \frac{y_o}{[P(y)]^{\alpha}}.$$

For example, if P(y) were equal to .001, meaning that .999 of all observations have incomes below this level,

$$y^* = \frac{y_o}{(.001)^{1/\alpha}}.$$

Substituting:

$$y^* = \frac{6{,}000}{\frac{1}{(.001)^{2.541}}} = \frac{6{,}000}{(.001)^{.3937}} = \$90{,}918.$$

Therefore, $90,918 would be the value that would be assigned to families in the .999 percentile for Tract 14.

For non-Negroes, running the equations using the data included in this section, yields these results:

For y_o = \$6,000, $\log P(y) = \underset{(11.16)}{.7131} (\log y_o - \log y)$ $R^2 = .87$

For y_o = \$7,000, $\log P(y) = \underset{(9.46)}{.801} (\log y_o - \log y)$ $R^2 = .82$

For y_o = \$8,000, $\log P(y) = \underset{(7.15)}{.877} (\log y_o - \log y)$ $R^2 = .66$

For y_o = \$9,000, $\log P(y) = \underset{(5.61)}{.511} (\log y_o - \log y)$ $R^2 = .79$

For y_o = \$10,000, $\log P(y) = \underset{(5.40)}{.565} (\log y_o - \log y)$ $R^2 = .80$

Again, $6,000 has the highest R^2 which indicates the best fit. Using the appropriate α, and the P(y) computed in a later step, the income for blocks with incomes greater than $50,000 can be calculated.

High values are generally the result of a gradual slope and/or few or no observations in many of the income classes included.

STEP 3 - Calculating the Mean Owner-Occupied Housing Value and Contract Rental Values for Total, Negro and Non-Negro

For owner-occupied units, this is done by taking the aggregate value of owner-occupied housing units on the block and dividing this number by the number of such units on the block. This is done directly for Total and for Negro.

Example: Block 105 in Tract 14 has an aggregate owner-occupied value equal to $30,750 and a total count of owner-occupied units of 5.

Dividing $30,750 by 5 gives a mean owner-occupied value of $6,150. The Negro value and count is equal to the total count giving the same mean owner-occupied value of $6,150. For non-Negroes the aggregate housing value of Negroes is subtracted from the total aggregate housing value giving non-Negro aggregate housing value. Then, the number of Negro owner-occupied units are subtracted from the total number of owner-occupied units to get the number of non-Negro owner-occupied units. The non-Negro aggregate owner-occupied value is then divided by the number of non-Negro owner-occupied units to get a mean value of owner-occupied housing for non-Negroes. Block 105 has no non-Negro owner-occupied units, so another example is presented.

Block 203 has a total count of 12 for owner-occupied units and an aggregate owner-occupied value of $89,750 given a mean $\frac{89,750}{12}$ of $7,479.15. The aggregate value of Negro owner-occupied units is $63,500; the count for these units is 9, so $63,500 divided by 9 equals $7,055.56, the mean Negro owner-occupied housing value. Subtracting $63,500 from $89,750 gives a non-Negro aggregate value of owner-occupied units of $26,250. Subtracting 9 from 12 gives a count of 3 for these non-Negro units and dividing $26,250 by 3 gives a mean of $8,750 for non-Negro owner-occupied units on this block.

Average contract rent is computed in exactly the same manner. Aggregate contract rent for a block is divided by the number of such units to get a mean. Negro figures are subtracted from total figures to get non-Negro figures. Block 105, of Tract 14, has 16 contract rental units and an aggregate contract rental value of $900. Dividing $900 by $16 gives a mean value of $56.25. All of these units are Negro-occupied so the Negro mean contract rental is also $56.25 and there are no non-Negro contract rental units for this block.

To illustrate the procedure for non-Negroes, Block 203 has 59 contract rental units with an aggregate value of $2,985. 57 of these units are Negro-occupied with an aggregate contract rental of $2,905. Subtracting 57 from 59 and $2,905 from $2,985 gives 2 units with an aggregate contract rental of $80. Dividing this gives a non-Negro average of $40.

For our Tract 14 example blocks we have:

TOTAL

	Owner-Occupied		Contract Rental	
	Number	Average Value	Number	Average Rental
Block 105	5	$6,150.00	16	$56.25
Block 203	12	7,479.17	59	50.59

NEGRO

	Owner-Occupied		Contract Rental	
	Number	Average Value	Number	Average Rental
Block 105	5	$6,150.00	16	$56.25
Block 203	9	7,055.56	57	50.96

NON-NEGRO

	Owner-Occupied		Contract Rental	
	Number	Average Value	Number	Average Rental
Block 105	-	-	-	-
Block 203	3	$8,750.00	2	$40.00

STEP 4* - Finding the Percentile Equivalents in the Tract Distribution of Median Owner-Occupied Housing Values and Median Contract Rental Value

This generally involves interpolations using the following procedure:

1. Find the interval in which the block housing value is located. Then take the percentage of housing units whose housing values are less than or equal to the upper value of this interval.

   Example:

   A. Looking at Table II, Block 105's Total Owner-Occupied Value of $6,150 falls in the $5,000 - $9,999 category, with 89.3% of the units having values equal to or less than $9,999.

   B. Looking at Table III, Block 105's Negro Owner-Occupied Value of $6,150 falls in the $5,000 - $7,499 category. 61.8% have values equal to or less than $7,499.

   C. From Table IV, Block 105's Total Contract Rental of $56.25 belongs in the interval $40 - $59.99. 69.2% have contract rentals less than or equal to $59.

   D. From Table V, Block 105's Negro Contract Rental of $56.25 belongs in the interval $50 - $59.99. 72.1% of the tract's Negro contract rental housing units have values less than or equal to $59.99.

2. Obtain the percentage of units in this interval. This is done by subtracting the cumulative percentage of housing units whose housing values are below the beginning value of this interval from the cumulative percentage of this interval.

---

*Only Block 105, Tract 14, is described in detail in the remaining steps.

Example:

A. For Tract 14, 25.9% of the Total owner-occupied units have values less than or equal to $5,000, the lower end of the interval. Subtracting 25.9% from 89.3% (from 1A) gives 63.4% in this interval.

B. For the Negro owner-occupied units of Tract 14, 27.0% have values equal to or less than $5,000, so 34.8% have values in this interval (61.8% - 27.0%).

C. For Tract 14's contract rental units, 12.7% in the tract have contract rentals less than or equal to $40, so that 56.5% are in this interval (69.2% - 12.7%).

D. For Tract 14's Negro contract units, 42.4% have values less than or equal to $50, leaving 29.7% in this interval (72.1% - 42.4%).

3. Subtract the block's actual housing value from the closing housing value of the interval in which it falls.

Example:

A. For Block 105's owner-occupied units, subtract $6,150, the observed value, from $9,999, the upper value of the interval, to get $3,849.

B. For Block 105's Negro owner-occupied units, subtract $6,150, the observed value, from $7,499, the upper value of the interval, to get $1,349.

C. For Block 105's total rental units, subtract $56.25, the observed value, from $59.99, the upper value of the interval, to get $3.74.

D. For Block 105's Negro rental units, subtract $56.25, the observed value from $59.99, the upper value of the interval, to get $3.74.

4. Find the size of the housing interval. This is done by subtracting the beginning value of the housing interval from the closing value of the housing interval.

Example:

A. Total owner-occupied units of Tract 14: Subtract $5,000 from $9,999 to get $4,999 as the size of this housing interval.

B. Negro owner-occupied units of Tract 14: Subtract $5,000 from $7,499 to get $2,499 as the size of the housing interval.

C. Total contract rental units of Tract 14: Subtract $40.00 from $59.99 to get $19.99 as the size of this interval.

D. Negro contract rental units of Tract 14: Subtract $50.00 from $59.00 to get $9.99 as the size of this interval.

5. Divide the number obtained in the third step by subtracting the block's actual housing value from the closing value of the interval by the size of the housing interval calculated in the fourth step.

Example:

A. For owner-occupied units of Block 105, $3,849 divided by $4,999 is .77.

B. For Negro owner-occupied units of Block 105, $1,349 divided by $2,499 is .54.

C. For contract rental units of Block 105, $3.74 divided by $19.99 is .187.

D. For Negro contract units of Block 105, $3.74 divided by $9.99 is .374.

6. Multiply the relationship obtained from Step 5 by the percentage of units in the interval.

Example:

A. For owner-occupied units, Block 105, multiplying .77 by 63.4% equals 48.82%.

B. For Negro owner-occupied units, Block 105, multiplying 154 by 34.8% equals 18.79%.

C. For renter-occupied units, Block 105, multiplying .19 by 56.5% equals 10.57%.

D. For Negro renter-occupied units of Block 105, multiplying .37 by 29.7% equals 11.11%.

7. Subtract the percentage found in Step 6 from the percentage of units with housing values less than or equal to the upper housing value from Step 1. This percentage is then assigned to the block housing value.

Example:

A. Subtracting 48.82% from 89.30% gives a percentile of 40.48% which is assigned to Block 105's owner-occupied mean total housing value.

B. Subtracting 18.79% from 61.8% gives a percentile of 43.01% which is assigned to Block 105's Negro average owner-occupied value.

C. Subtracting 10.57% from 69.2% gives a percentile of 58.63% which is assigned to Block 105's total contract rental value.

D. Subtracting 11.11% from 72.1% gives a percentile of 60.99% which is assigned to the Negro average contract rental of Block 105.

E. Following this procedure for Block 203 the following percentiles are obtained:

	Total	Negro	Non-Negro
Owner-Occupied	57.34	55.61	40.48
Contract Rental	42.63	45.25	17.65

Note, if the value is within the first interval, then the procedure is still the same, but the percentage of units in the interval is equal to the percentage in the first interval and the denominator in the procedure is the upper or positive value of the interval. If, instead, the value is in the highest interval the values used to close the interval are those commonly used by the Census, $350 for rentals, $60,000 for owned housing.

STEP 5 - Finding the Normal Deviate Values

For each of these percentiles, find the corresponding normal deviate values by going into the normal curve area table with the percentiles and finding the normal deviate arguments. When the percentile is 50 or over the procedure is direct. When it is less than 50, subtract the percentile from 100 and then go into the normal curve area table. In this case, the value has a negative sign.

Another approach, and the one used here in cases where the value is less than 50, is to subtract the arguments in the table from 1.00 and to do the calculations directly. The negative sign is still used for percentiles less than 50. Total, Negro, and non-Negro are all done in exactly the same way using the same table.

Example:

A. Block 105's owner-occupied housing percentile is divided by 100 to get .4048--the decimal or probability form. Then, looking at the converted normal deviate table, for values less than .50, the .4048 is located between .4052 and .4013, which are equivalent in the table to .24 and .25, respectively. Since .4048 is between these percentiles but not equivalent to either one, it is again necessary to interpolate in a manner similar to that used in Step No. 3.

   1. The interval is located .4052 to .4013. These are assigned values of .24 and .25, so that the interval size is .01.
   2. .4048, the block percentile, less .4013, the end of the interval furthest from the median, equals .0035.
   3. .4052, the nearer end of the interval, less .4013, the further end of the interval, equals .0039, the probability size of the interval.
   4. .0035 divided by .0039 equals .897.
   5. .897 times .01, the size of this interval, equals .0090.

6. .25 minus .0090 equals .241 which is the argument assigned to .4048, Block 105's owner-occupied housing percentile. A negative sign is attached to .241 because the probability value is below .50.

B. Block 105's Negro's owner-occupied housing percentile is 43.01.

1. Going to the converted normal deviate table, .4301 is located between .4325 and .4286. They are equivalent to .17 and .18, respectively, so that again the interval size is .01.
2. The block percentile value less than percentile more distant from median (.4301 less .4286) equals .0015.
3. Subtracting the percentile of the more distant interval end from the nearer interval end, .4325 less .4286 equals .0039.
4. Dividing .0015 by .0039 equals .38.
5. .38 times .01, the size of the interval, equals .0038.
6. .18 less .0038 equals .1762 which is the normal deviate value assigned to Block 105's Negro owner-occupied housing percentile. A negative sign is attached to .1762 because the probability is below .50.

C. Block 105's total contract rental has a percentile value of 58.63 assigned to it. This is above 50 percent so that the direct procedure may be used:

1. .5863 falls between .5832 and .5871. Going to the normal deviate table they are equivalent to .21 and .22, respectively, so that again the interval size is .01.

2. Subtracting the block's percentile value from the percentile of the far end of the interval, .5871 - .5863 yields .0008.
3. Subtracting the value of the near end of the interval from the far end yields .0039 (.5871 - .5832).
4. .0008 divided by .0039 equals .205.
5. .205 times .01 equals .0021.
6. .22 less .0021 equals .2179, the normal deviate value assigned to Block 105's average contract rental.

D. Block 105's Negro contract rental percentile is 60.99. Since this is 50 percent or above, the procedure directly uses the normal deviate table.

1. Going to the normal deviate table .6099 falls between .6064 and .6103. These are equivalent to .27 and .28 in the table, respectively, and there is .01 between them.
2. .6103, the percentile at the end of the interval, less .6099, the block percentile, equals .0004.
3. .6103, the percentile at the end of the interval, less .6064, the percentile at the beginning of the interval, equals .0039.
4. .0004 divided by .0039 equals .103.
5. .103 times .01 equals .0010.
6. .28 less .0010 equals .279, the value in the normal deviate table assigned to Block 103's Negro contract rental percentile value.

So the normal deviate values for Tract 14, Block 105 are:

	Total	Negro
Owner-Occupied	-.241	-.1762
Renter-Occupied	.2179	.279

STEP 6 - Computing r for Owner-Occupied-Income and Renter-Occupied-Income Relationships

The relationship between rental values and income for the total population and by race, and owner-occupied housing and income for the total population and by race, are computed using the formula for a correlation coefficient:

$$r = \frac{\Sigma x_i y_i}{(\Sigma x_i)^2 \, \Sigma (y_i)^2}$$

where

$$x_i = X_i - \bar{X},$$

the difference between the actual observed income and the mean income for the group and

$$y_i = Y_i - \bar{Y};$$

the difference between the actual observed housing value and the mean housing value for this group. This can, of course, be done directly by running a linear regression, and obtaining the r or correlation between these two variables.

Basically this answers the question of whether or not these variables move together--are they "co-related" and by how much? The value of r goes between 0 and +1 or -1. If r equals either -1 or +1 all observations lie on a straight line and the relationship is said to be perfect. If r = 0, there is no discernible linear relationship between the two variables, and so the variables are uncorrelated.

The r's or correlation coefficients are computed from the data items in the 4th Count Census of Housing based on the 20 percent sample. The data items used are "Value, Income, Tenure, and Race of Head," which give a count of owner-occupied units for which value is tabulated, cross classifying by race 7 value categories of housing by 7 income categories. From this it is possible to compute the correlation between income and owner-occupied housing value for total and for Negroes. The Negro count is again subtracted from the total count to get the non-Negro count. The second group of data items are "Gross Rent, Income, Tenure, and Race of Head," which cross classify by race 8 categories of monthly gross rent by 8 categories of income. From this it is possible to compute the correlation between rentals and income for Total and Negro. (The Negro count is subtracted from the total count to obtain the non-Negro count.)

The steps of this procedure are:

1. Find the mean value of income (for all renters, for example).
2. Find the mean housing value for this group (mean gross rent for this example).
3. For each observation on income subtract the mean from the actual value--compute the deviations from the mean.
4. For each observation on housing also compute the deviations from the means by subtracting the mean from the actual values.
5. Multiply the deviations between actual and mean income values from 3 by the deviations between actual and mean housing value from 4.
6. Add up the terms found in 5 over the group being considered (all renters for this example) to obtain a measure of the association between housing and income.
7. Square each deviation value found in 3 and 4--the differences between actual and mean values.
8. Add up the squared values of deviation between income and mean income.
9. Add up the squared values of the deviation between housing value and mean housing value.

10. Multiply the summed squared deviation values from 8 by the summed squared deviation values from 9.

11. Take the square root of these sums from 10 to get a geometric mean of the variable variances.

12. Divide the measure of association from 6 by the geometric mean of the variables respective variances from 11. This is the r for this group.

Example: Table XI gives the data for Tract 14 on the relationship between income and owner-occupied value while Table XII gives the count between income and rentals for Tract 14. The midpoints of each closed category are used while the lowest category of owner-occupied values is assigned $3,500, the value the Census uses for this category. The highest housing value is assigned $60,000, again a value used by the Census. The value used for the highest income class is specific to each racial group or total and is computed based on a method described below. The rentals are also assigned midpoints of closed categories, and Census numbers for the other categories: $25 for the lowest category and $350 for the highest category.

For tracts with observations in the $25,000+ range in the income-housing cross-classification, a figure is needed for use in calculating the r between income and housing. The procedure followed is similar to that used in closing the open-ended class of the tract income distribution. A Pareto distribution is assumed, y_o, the beginning level of the distribution is set equal to $10,000 (closest value in distribution to city-wide median) and the parameter of the distribution, α, is calculated by the formula of the Pareto distribution, $p(y) = \left(\frac{y_o}{y}\right)^{\alpha}$ in order to find the median for each $25,000+ class.

For Tract 14, there are no renters with incomes above $25,000 in the cross-classification so this procedure is followed only for owners. Looking

TABLE XI

1970, PHILADELPHIA, TRACT 14: INCOME BY OWNER-OCCUPIED VALUES

Total Owner-Occupied Values (Dollars)	INCOME CLASSES (DOLLARS)						
	< 3,000	3,000 - 4,999	5,000 - 6,999	7,000 - 9,999	10,000 - 14,999	15,000 - 24,999	25,000 +
< 5,000	46	9	19	18			7
5,000 - 9,999	89	66	26	27	13	3	
10,000 - 14,999	15	7		7	4		
15,000 - 19,999	5		5	4		7	
20,000 - 24,999							
25,000 - 34,999							
35,000 +							
Negro							
< 5,000	46	9	19	18			4
5,000 - 9,999	83	66	21	27	13	3	
10,000 - 14,999	15	7		7	4		
15,000 - 19,999	5		5	4			
20,000 - 24,999							
25,000 - 34,999							
35,000 +							
Non-Negro							
< 5,000							3
5,000 - 9,999	6		5				
10,000 - 14,999						7	
15,000 - 19,999							
20,000 - 24,999							
25,000 - 34,999							
35,000 +							

Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Housing Summary, Philadelphia county.

TABLE XII

1970, PHILADELPHIA, TRACT 14: INCOME BY GROSS RENTALS

Rental Intervals (Dollars)	Income Classes (Dollars)							
	< 2,000	2,000 - 2,999	3,000 - 4,999	5,000 - 6,999	7,000 - 9,999	10,000 - 14,999	15,000 - 24,999	25,000 +
Total								
< 40	38	10	21	-	6	-		
40 - 59	271	98	122	51	29	6		
50 - 79	124	122	184	103	63	26		
80 - 99	60	74	89	26	53	23		
100 - 149	24	5	27	16	32	6	9	
150 - 199	-	-		5				
200 +	-							
Negro								
< 40	38	10	15	-	6			
40 - 59	271	98	116	51	29	6		
60 - 79	124	122	184	103	58	26		
80 - 99	60	74	78	26	47	23		
100 - 149	24	5	27	46	32	6	9	
150 - 199	-			5				
200 +	-							
Non-Negro								
< 40			6					
40 - 59			6					
60 - 79					5			
80 - 99			11		6			
100 - 149								
150 - 199								
200 +								

Source: U. S. Bureau of the Census, Dept of Commerce, 1970 Census, 4th Count Housing Summary, Philadelphia county.

at Table XI, form the reverse income distribution for total, Negro and non-Negro from $10,000 up. (The cumulative distribution of the Pareto distribution is reversed.) This is done by adding up all observations in each income category to get the count, adding up the interval counts in reverse to get the cumulative count and then dividing the cumulative count of each income category by the total count to obtain the reverse cumulative percentile distribution:

Total

Income Interval	Count	Reverse Cumulative Count	Reverse Percentage Distribution
$10,000 - 14,999	17	34	1.0
$15,000 - 24,999	10	17	.5
$25,000 +	7	7	.21

Negro

Income Interval	Count	Reverse Cumulative Count	Reverse Percentage Distribution
$10,000 - 14,999	17	24	1.0
$15,000 - 24,999	3	7	.29
$25,000 +	4	4	.17

Non-Negro

Income Interval	Count	Reverse Cumulative Count	Reverse Percentage Distribution
$10,000 - 14,999	0	10	1.0
$15,000 - 24,999	7	10	1.0
$25,000 +	3	13	.3

The procedure for getting a Pareto distribution is to run a linear regression in logs suppressing the constant term. Therefore, next, take logs of the beginning value of each income interval and the cumulative percentiles, and subtract the log of $10,000, the beginning value of the Pareto distribution:

		Cumulative Distributions		
Incomes		Total	Negro	Non-Negro
(log $10,000 - log $10,000)	0.0	0.0	0.0	0.0
(log $10,000 - log $15,000)	-0.18	-0.30	-0.54	0.0
(log $10,000 - log $25,000)	-0.40	-0.68	-0.77	-0.52

Fit these logs to the equation

$$\log P(y) = \alpha(\log y_o - \log y)$$

by running a linear regression suppressing the constant term. The results for Tract 14 are:

Total $P(y)_T = 1.722 (\log y_o - \log y)$ adj. $R^2 = .999$

Negro $P(y)_N = 2.133 (\log y_o - \log y)$ adj. $R^2 = .904$

Non-Negro $P(y)_{NN} = 1.099 (\log y_o - \log y)$ adj. $R^2 = .754$

Next, use y_o = $10,000 and computed α's to get a median for the open-ended classes by setting P(y) equal to one-half of the cumulative percentile in the $25,000+ class solving the equation

$$y_i^* = \frac{\$10,000}{P(y)^{1/\alpha}} \qquad i = T, N, \text{ or } NN$$

For Total

$$y_T^* = \frac{10,000}{\frac{.21}{2}^{\frac{1}{1.722}}} = \frac{10,000}{(.105)^{\frac{1}{1.722}}} = \frac{10,000}{.2701} = \$37,018$$

For Negroes

$$y^{*}_{N} = \frac{10,000}{\frac{1}{\frac{.17}{2}^{2.133}}} = \frac{10,000}{\frac{1}{(.085)^{2.133}}} = \frac{10,000}{.3148} = \$31,762$$

For Non-Negroes

$$y^{*}_{NN} = \frac{10,000}{\frac{1}{\frac{.13}{2}^{1.099}}} = \frac{10,000}{\frac{1}{(.15)^{1.099}}} = \frac{10,000}{.1779} = \$56,194$$

The r's for Tract 14 were then computed from the cross-classified data of the Tables XI and XII and estimated medians for the $25,000+ class. Using a linear regression for the r between owner-occupied values and income, tne observations cross classifying the highest income value with the lowest owned value result in low or negative correlations. For total, the r between owned housing and income is .0635 calculated by using the $3,500 for the lowest owner-occupied value and $60,000 for the highest owned value midpoints of each closed category, $1,500 for the lowest income classes and $37,018 for the highest income class. For non-Negroes and Negroes, the same values are used except for the highest income. For Negroes $31,762 is used, while for non-Negroes $56,194 is used.

The r for Negro and non-Negro owned values are both negative so the total value .0635 is used. The r's between renter-occupied and income are .3149 for total, .3161 for Negroes, and .3422 for non-Negroes.

NOTE: Correlation coefficients are only computed for the 1970 block income estimates, since the Census did not make available the cross-calculations of income by housing information in 1960. Instead, for 1960, a standard r of .3 is used as an adjustment factor in the housing income relationship. This is based on a number of studies including Tong Hun Lee,

"Demand for Housing: A Cross-Sectional Analysis."[1] Using the 1958 Survey of Consumer Finances of the University of Michigan, he found the r^2 relating to cost of home purchased by income and other socio-economic variables to be .26. Alan Winger in a 1968 study[2] used FHA data to look at the income-housing relationship. For 1962-64 home purchases he found the relationship between value of home purchased and income on an SMSA unit basis, as measured by a log-log regression, had an r^2 of .55 for all units, .47 for new units, and .68 for existing units.

Maisel and Winnick[3] in a study including only families with incomes between $2,000 and $10,000, found r^2's ranging from .1 to .9 using a 1950 study of Consumer Expenditures. The higher r^2's are for weighted grouped data by cities while the lower r^2's are for individual data. These patterns are true for renters and owners as can be seen in the accompanying chart.

Geoffrey Carliner, in a 1973 study on income elasticity of housing demand including both owners and renters, got an R^2 of .32 using a permanent income concept and certain demographic variables, on BLS household data.[4]

Thus, to summarize the available evidence: The correlation between housing and income for small groups should fall between the .5-.9 of large grouped data and .1 of individual data; Carliner's results show an R^2 of

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[1] Review of Economics and Statistics, Vol. 45, No. 2 (May 1963) pp. 190-196.

[2] Alan Winger, "Housing and Income," Western Economic Journal, Vol. 6, No. 3 (June 1968) pp. 226-232.

[3] Sherman J. Maisel and Louis Winnick, "Family Housing Expenditures: Elusive Laws and Intrusive Variances," Conference on Consumption and Savings I, eds. Friend and Jones (Philadelphia: Wharton School of Finance) 1960, pp. 374-5.

[4] Geoffrey Carliner, "Income Elasticity of Housing Demand," Review of Economics and Statistics, Vol. 55, No. 4, Nov. 1973, 528-32.

COMPARISON OF r^2 FROM GROUPED AND UNGROUPED DATA

Subgroups	Number of Observations	Grouped Data r^2	Individual Data r^2
Owners without mortgages			
Large northern cities and suburbs	697	.843	.111
Owners with mortgages			
Large northern cities and suburbs	1,055	.907	.113
Renters			
Large northern cities and suburbs	1,760	.926	.108

Source: Maisel and Winnick, op. cit., p. 390.

.32, Lee's .26. Our average 1970 estimates are also in the .2-.3 range. Based on these findings, an r of .3 seemed appropriate to use.

STEP 7 - Using the r's to Adjust the Percentiles

These r's are now used to adjust the percentiles for the imperfect relationship between income and housing. If the r were to equal 1 the relationship would be perfect and no such adjustment needed, while if it were 0, the mean income value would be used instead since housing would then contain no relationship to income.

The arguments from the normal deviate table are multiplied by the r to make the adjustment.

Example:

A. Tract 14, Block 105's argument for owner-occupied housing values overall is -.241 while the r between income and owner-occupied value for Tract 14 is .064. -241 times .064 equals -.0154, the adjusted argument.

B. Tract 14, Block 105's Negro owner-occupied housing has an argument of -.1762 and the r between Negro income and owner-occupied values in Tract 14 is .064. -.1762 times .064 equals the adjusted argument -.0112.

C. Tract 14, Block 105's contract rental units have an argument of .2179 while Tract 14 has an r of .315 between contract rentals and income. .2179 times .315 equals .069, the adjusted argument.

D. Tract 14, Block 105's Negro contract rental argument is .279 while the r between Negro incomes and rents in Tract 14 is .316. .316 times .279 yields .088, the adjusted argument.

STEP 8 - Finding the Percentiles from the Normal Deviate Tables

The next step is to find the percentile in the normal deviate tables that correspond to the adjusted arguments computed in Step 7. Generally, this also involves interpolations.

Example:

A. Block 105's total owner-occupied units have an adjusted argument of -.0154.

1. Find the interval in which the adjusted argument falls: -.0154 is between .01 and .02, which correspond to .4960 and .4920, respectively, in the table for percentiles below the 50th percentile.
2. The size of the interval between the two percentiles .4960 and .4920 is .0040.
3. .02, the more distant end of the interval, less the adjusted argument, .0154, equals .0046.
4. The distance within the entire interval is .01.
5. .0046 divided by .01 yields .46.
6. .46 times .0040 equals .0018, but, because this is below the 50th percentile, it has a minus sign.
7. .4920 less (-.0018) equals .4938, the adjusted percentile.

B. Block 105's Negro owner-occupied units have an adjusted argument of .0112.

1. .0112 falls in the interval .01 to .02, which correspond to the percentiles .4960 and .4920, respectively, in the table for percentiles below the 50th percentile.

2. The size of the interval between these percentiles is .0040.
3. .02, the more distant end of the interval, less .0112 equals .0088.
4. The size of the interval in the normal deviate table is .01.
5. .0088 divided by .01 equals .88.
6. .88 times .0040, the percentile interval, equals .0035.
7. .4920 less (-.0035) yields .4955, the adjusted percentile.

C. Block 105's contract rental units have an adjusted argument of .069 which is positive.
1. .069 falls in the interval .06 to .07 which correspond to .5239 and .5279, respectively.
2. The size of the interval between the two percentiles is .0040.
3. .07, the more distant end of the interval, less .069 equals .001.
4. The distance between the two ends of the interval is .01.
5. .001 divided by .01 equals .1.
6. .1 times .0040, the percentile interval, equals .0004.
7. .5279 less .0004 is .5275, the adjusted percentile for contract rental units.

D. Block 105's Negro contract rental units have an adjusted argument equal to .088.
1. .088 falls between .08 and .09, which correspond to .5319 and .5359, respectively.
2. The size of the interval between these two percentiles is .0040.

3. .09, the more distant end of the interval, less .088 is .002.
4. The size of the interval is .01.
5. .002 divided by the size of the interval, .01, is .2.
6. .2 times .004 is .0008.
7. .5359 less .0008 yields .5351, the adjusted percentile.

STEP 9 - Finding the Income Numbers Associated with the Owner-Occupied and Rental Housing Adjusted Percentiles

Step 9 is finding the income figure associated with the adjusted percentiles. Once the adjusted percentiles have been computed and the income distributions closed, the next step is to find the income that corresponds to the adjusted percentile for each housing racial group. In order to do this we take the percentile and go to the corresponding income distribution. Since, in general, the percentile will not correspond exactly to the breakdowns in the income distribution, it is necessary to interpolate.

Example:

A. 1. Take Block 105's total owner-occupied percentile of .4938. .4938, in percentage terms (multiply by 100), falls between the cumulative percentages 38.3% to 50.9% and the corresponding income interval of $4,000 to $4,999.
2. Next, find the amount of income included in this category and the percentile size of the interval. The dollar amount is $999, and the percentage of observations in this range is 12.6%
3. Then, compute the share of the interval that should be subtracted from the income end of the interval in percentile terms. In this case 50.9% less 49.38%, the

observed percentile, equals 1.52%. 1.52% divided by 12.6%, the percentile size of the interval, is equal to .121.

4. Convert this into dollar terms by multiplying this percentage by the amount of income in the interval. .121 times the dollar size of the interval, $999, equals $121.
5. Then, subtract this from the highest income included in this income interval, $4,999, to get the income equivalent to the 49.38 percentile. $4,999 less $121 equals $4,878, the income value assigned to Block 105's owner-occupied units.

B. 1. Take .4955, the adjusted percentile of Block 105's Negro owner-occupied units. .4955, in percentile terms, falls between the cumulative percentages of 38.1% to 51.1%. The corresponding income interval is $4,000 to $4,999.
2. The amount of income included in this category is $999 while the percentage of observations in this range is 13%.
3. 51.1%, the upper end of the interval, less 49.55, the observed percentile, is 1.55%. 1.55% divided by 13%, the percentile size of the interval, is equal to .119.
4. Multiplying this percentage, .119, times $999 equals $119.
5. $4,999, the highest income in interval, less $119 equals $4,880, the income value assigned to Block 105's Negro owner-occupied units.

C. 1. Take .5275, the adjusted percentile for Block 105's contract rental units. .5275 in percentage terms falls between the cumulative percentages 50.9% and 59.6%. The corresponding income interval is $5,000 to $5,999.

2. The amount of income included in this class is $999 while the percentage of observations is 8.7%.

3. 59.6%, the upper end of the interval, less 52.75%, the observed percentile, is 6.85%. 6.85% divided by 8.7%, the percentile size of the interval, is equal to .79.

4. Multiplying this percentage, .79 times $999, equals $789.

5. $5,999, the highest income interval, less $789 equals $5,210, the income value assigned to Block 105's contract rental units corresponding to the 52.75%.

D. 1. Take .5351, the adjusted Negro contract rental percentage for Block 105. In percentage terms this falls between 51.1% to 60.0%. The corresponding income interval is $5,000 to $5,999.

2. The amount of income included in this class is $999 while the percentage of observations is 8.9%.

3. 60.0%, the upper end of the interval, less 53.51%, the observed percentile, is 6.49%, while 6.49% divided by 8.9%, the percentile size of the interval, is equal to .73.

4. Multiplying the percentage .73 times $999, the income size of the interval, equals $728.

5. $5,999, the highest income in interval, less $728 equals $5,271, the income value assigned to Block 105's Negro contract rental units corresponding to the 53.51%.

E. If a block has an adjusted percentile in the income range 50,000+, the Pareto distribution calculated in Step 2 is used to calculate the corresponding income.

1. First, calculate the appropriate cumulative percentile. This is done by taking the block percentile and the percentile associated with the y_o of the tract's Pareto distribution. Then solve the equation

$$P(y) = 1 - \frac{PB - Py_o}{(1 - Py_o)}$$

Where $P(y)$ = adjusted cumulative percentage of block;

PB = adjusted percentage of block;

Py_o = cumulative percentage of y_o used in fitting the Pareto distribution to tract income distribution.

Example: Block 202 non-Negro renters have an adjusted percentile of 89.73%, or .8973, in decimal form, so this will be used to illustrate the method. In Step 2, the Pareto distribution fitted to Tract 14 non-Negroes had parameters of $\alpha = .7131$ and $y_o = 6,000$. 46.4%, or .464, in decimal form, is the cumulative percentage associated with $6,000 for Tract 14 Non-Negroes (from Table I). Solving for $P(y)$:

$$1 - \frac{.8973 - .4640}{1 - .4640} = 1 - \frac{.4333}{.536} = .1916$$

2. Next, solve the Pareto distribution for the adjusted cumulative percentage of the block by solving the formula:

$$y^* = \frac{y_o}{[P(y)]^{1/\alpha}}$$

This y^* is then the income assigned to the block's housing value. Example: Using the information calculated above:

$$y^* = \frac{6,000}{(.1916)^{\frac{1}{.7131}}} = \$60,879$$

so that $60,879 is the income value assigned to Tract 14, Block 202, Non-Negro Renters.

STEP 10 - Calculating Adjustments for Different Relationships between Owner Value and Renter Value of Housing and Income

This step calculates owner and renter adjustments using the cross-classified information on rental values by income and owned values by income.

Example:

A. For each tract form the cumulative income distribution of renters, owners, and then renters and owners combined. Combine the two lowest renter categories, < $2,000, and $2,000 - $3,000, to have consistent income categories for the three distributions. This is done for Tract 14 in Tables XIII-XV. In each distribution the count is added over the income interval to get the cumulative count. The cumulative count of each interval is then divided by the total count to get the cumulative percentage distribution.

TABLE XIII

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION--OWNERS

	Total			Negroes			Non-Negroes		
	Count	Cumulative Count	Cumulative Percentile Distribution	Count	Cumulative Count	Cumulative Percentile Distribution	Count	Cumulative Count	Cumulative Percentile Distribution
< $3,000	155	155	41.1	149	149	41.9	6	6	28.6
$ 3,000 - $ 4,999	82	237	62.9	82	231	64.9	0	6	28.6
$ 5,000 - $ 6,999	50	287	76.1	45	276	77.5	5	11	52.4
$ 7,000 - $ 9,999	56	343	91.0	56	332	93.3	0	11	52.4
$10,000 - $14,999	17	360	95.5	17	349	98.0	6	11	52.4
$15,000 - $24,999	10	370	98.1	3	352	98.9	7	18	85.7
$25,000 +	7	377	100.0	4	356	100.0	3	21	100.0

Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Housing Summary Tape, Philadelphia county.

TABLE XIV

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION--RENTERS

	Total			Negroes			Non-Negroes		
	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage
< $2,000	517	517	30.0	517	517	30.6	0	0	0
$ 2,000 - $ 2,999	309	826	47.9	309	826	48.9	0	0	0
$ 3,000 - $ 4,999	443	1,269	73.7	420	1,246	73.8	23	23	67.6
$ 5,000 - $ 6,999	201	1,470	85.3	201	1,447	85.7	0	23	67.6
$ 7,000 - 9,999	183	1,653	95.9	172	1,619	95.9	11	34	100.0
$10,000 - $14,999	61	1,714	99.5	61	1,680	99.5	0	34	100.0
$15,000 - $24,999	9	1,723	100.0	9	1,689	100.0	0	34	100.0
$25,000 +	0	1,723	100.0	0	1,689	100.0	0	34	100.0

Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Housing Summary Tape, Philadelphia county.

TABLE XV

1970, PHILADELPHIA, TRACT 14: INCOME DISTRIBUTION--RENTERS AND OWNERS COMBINED

	Total			Negro			Non-Negro		
	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage	Count	Cumulative Count	Cumulative Percentage
< $3,000	981	981	46.7	975	975	47.7	6	6	10.9
$ 3,000 - $ 4,999	525	1,506	71.7	502	1,477	72.2	23	29	52.7
$ 5,000 - $ 6,999	251	1,757	83.7	246	1,723	84.3	5	34	61.8
$ 7,000 - $ 9,999	239	1,996	95.0	228	1,951	95.4	11	45	81.8
$10,000 - $14,999	78	2,074	98.8	78	2,029	99.2	0	45	81.8
$15,000 - $24,999	19	2,093	99.7	12	2,041	99.8	7	52	94.5
$25,000 +	7	2,100	100.0	4	2,045	100.0	3	55	100.0

Source: U. S. Bureau of the Census, Dept. of Commerce, 1970 Census, 4th Count Housing Summary Tape, Philadelphia county.

B. For each of these three income distributions obtain the incomes at the 25th, 50th, and 75th percentiles. This is done for the total population, and Negroes, and non-Negroes if the data permit.

The procedure employed is the interpolation method used throughout the program. If the procedure is applied to owners, these are the steps:

1. a. For the total population of owners, the 25th percentile is located within the less than $3,000 interval with 41.1% of the cumulative distribution. Because it is in the first interval the procedure is simplified.

   b. The dollar size of the interval is $2,999, the percentage size, 41.1%.

   c. The 25th percentile divided by the size of the interval, 41.1%, is .608.

   d. .608 times the dollar amount of the interval is $1,823.39, the income value assigned to the 25th percentile for owners of the overall population.

2. a. For the total population of owners, the 50th percentile falls in the 41.1%-62.9% interval of the cumulative distribution with income values of $3,000-$4,999.

   b. The dollar size of the interval is $1,999, and the percentile size is 21.8%.

   c. 62.9% less 50% equals 12.9%.

d. 12.9% divided by 21.8% equals .592. .592 times $1,999, the dollar amount of the interval, equals $1,183.40.

e. $4,999, the end of this interval, less $1,183.40 equals $3,815.60, the income value assigned to the 50th percentile of owners of the overall population.

C. 1. For the total population of owners, the 75th percentile falls between 62.9% and 76.1%, with associated income values of $5,000 and $6,999.

2. The dollar size of the interval is $1,999; the percentile size is 13.2%.

3. 76.1% less 75% equals 1.1%. 1.1% divided by 13.2% is .083.

4. .083 times $1,999, the dollar size of the interval, is $165.92.

5. $6,999, the end of the interval, less $165.92 equals $6,833.08, the income value assigned the 75th percentile of Tract 14 owners.

D. The same steps, if applied to renters and renters and owners together, yield the results presented in Table XVI.

E. The same steps, if applied to the subsets of Negro owners and renters, yield these results:

Percentiles	Renters	Owners	Renters and Owners
25th	$1,532.49	$1,790.40	$1,571.48
50th	3,087.96	3,703.65	3,187.91
75th	5,201.90	6,603.20	5,461.77

TABLE XVI

1970, PHILADELPHIA, TRACT 14: RENTERS AND RENTERS AND OWNERS INCOME AT THE 25TH, 50TH, AND 75TH PERCENTILES

	1	2	3	4	5	6
	Interval	Size of Interval	End of Interval Minus Percentile Being Computed	Column 3 Divided by Percentile Size of Interval	Column 4 Times Dollar Size of Interval	Computed Income = Dollar End of Interval Less Column 5
Renters: Total Population						
25%	< $3,000 0 - 47.9%	$2,999 47.9%	22.9%	.478	$1,433.52	$1,565.48
50%	$3,000 - $4,999 47.9% - 73.7%	$1,999 25.8%	23.7%	.919	1,837.08	3,161.92
75%	$5,000 - $6,999 73.7% - 85.3%	$1,999 11.6%	10.3%	.888	1,755.11	5,223.89
Owners & Renters: Total Population						
25%	< $3,000 0 - 46.7%	$2,999 46.7%	21.7%	.465	$1,394.54	$1,604.46
50%	$3,000 - $4,999 46.7% - 71.7%	$1,999 25.0%	21.7%	.868	1,735.13	3,263.87
75%	$5,000 - $6,999 71.1% - 83.7%	$1,999 12.0%	8.7%	.725	1,449.27	5,549.73

F. The same steps, if applied to the subsets of non-Negroes owners and renters, yield these results:

Percentiles	Renters	Owners	Owners and Renters
25th	$3,739.63	$ 2,621.13	$3,803.60
50th	4,479.26	6,797.10	4,869.06
75th	7,683.77	21,789.32	8,979.34

G. The next step is to divide the renter's income and owner's income at each of the three percentiles by the combined renter's and owner's income for that percentile.

For the total population at the 25th percentile, we have an income of $1,823.39 for owners, $1,565.48 for renters, and $1,604.46 for owners and renters combined. For the owners adjustment at the 25th percentile, divide $1,823.39 by $1,604.46 to obtain 1.136. For renters, divide $1,565.48 by $1,604.46 to get 0.976.

For the total population at the 50th percentile divide $3,815.60, the owners income at the 50% percentile, by $3,263.87, the renters and owners combined income at the 50% percentile, to get 1.169. Then, divide $3,161.92, the renters income at the 50% percentile, by $3,263.87 to get .969.

For the total population at the 75th percentile divide $6,833.08, the owners income at the 75% percentile, by $5,549.73, the combined income at the 75% percentile, to get 1.231. Then, divide $5,223.89, the renters income at the 75% percentile, by $5,549.73 to get .911.

For the subsets the adjustment figures are:

	NEGROES			NON-NEGROES	
	Renters	Owners		Renters	Owners
25th	.975	1.139	25th	.983	.689
50th	.969	1.162	50th	.920	1.396
75th	.952	1.209	75th	.856	2.427

STEP 11 - Obtaining Income Estimates Using Adjustments for Different Relationship between Owner Value and Renter Value of Housing and Income

The next step is to apply the renters and owners adjustment factors to the estimated block renter and block owner incomes. For blocks with incomes estimated at percentiles less than or equal to 25%, use the adjustment of the 25th percentile. For blocks with percentiles 75 or above, use the adjustment factor of the 75th percentile, while for blocks between the 25th and 75th (except exactly at the 50th) it is necessary to interpolate to get the appropriate factor.

Example:

A. The total block income of Block 105 owners, before adjusting, is $4,878, equivalent to the 49.38% percentile. The adjustment used, therefore, is between the adjustment factor of the 25th percentile for total owners, 1.136, and the adjustment factor of the 50th percentile for total owners, 1.169. Following the interpolation procedure applied throughout:

1. The interval size is 25% (50% - 25%), while 1.169, the adjustment factor of the 50th percentile, less 1.136, the adjustment factor of the 25% percentile, is .033.
2. 50% less 49.38% is .62%.
3. .62% divided by 25% is .0248.

4. .0248 times .033, the difference between the adjustment factors, equals .0008.
5. 1.169 less .0008 equals 1.1682, the adjustment factor used for the 49.38% of owners.
6. Multiply $4,878, the unadjusted owner income, by the adjustment factor, 1.168, to get the adjusted income for Block 105's owners, $5,698.

B. In order to adjust Block 105's Negro owner income of $4,880 equivalent to the 49.55% percentile, it is again necessary to interpolate between the adjustment factor of the 25th and 50th percentiles.

1. The 25th percentile adjustment factor for Negro owners of Tract 14 is 1.139, while the adjustment factor for the 50% is 1.162. 50% less 25% is 25%, and 1.162 less 1.139 is .023.
2. 50% less 49.55% is .45%.
3. .45% divided by 25% is .018.
4. .018 times .023 is .0004.
5. 1.162 less .0004 is 1.1616, the adjustment factor for the 49.55% of Tract 14 Negro owners.
6. Applying this to the income number, $4,880, yields $5,668, the adjusted income number assigned to Block 105's Negro owners.

C. Block 105's total renters have an income of $5,210, before adjusting, equivalent to the 52.75%. In order to adjust this income, it is necessary to interpolate between the 50% and 75% adjustment factors.

1. The size of the interval (75%-50%), is 25% while the adjustment factors are .969 at the 50% and .911 at the 75%. .911 less .969 is -0.058.
2. 75% less 52.75% is 22.25%.
3. 22.25% divided by 25% is .89.
4. .89 times -.058 equals -0.0516.
5. .911 less (-.0516) is .9626, the adjustment factor used for Tract 15, Block 105's renters.
6. Applying this adjustment figure, .9626, to $5,210 yields $5,015, the adjusted income figure for Block 105 renters.

D. Block 105's Negro renters have an income, before the renter adjustment, of $5,271 corresponding to the 53.51%. Since 53.51% falls between 50% and 75% it is again necessary to interpolate.

1. The size of the interval in percentage terms is 25% (75% less 50%). The adjustment factor at the 50% is .969, while it is .952 at the 75% for Negro renters. .952 less .969 is -.017.
2. 75% less 53.51%, the block's percentile, is 21.49%.
3. 21.49% divided by 25%, the percentile size of the interval, is .8596.
4. .8596 times -0.017, the difference between the adjustment factors, is -.0146.
5. .952 less (-.0146) equals .9666, the adjustment factor used for Block 105's renters.

6. Applying this to the income estimate, $5,271, yields $5,095, the adjusted income assigned to the Negro renters of Block 105, Tract 14.

STEP 12 - Calculating Weighted Average of Estimates of Renters' Income and Owners' Income

The last step in the procedure is to combine the renter's income and owner-occupied income, using weights to obtain an average income figure. The number of owner-occupied units and the number of renter units are used as the weights.

A. Find the total number of owner-occupied units for the block and the total number of renter units for the block. Add these together.

Example: For the average income of all residents of Block 105, the number of owner-occupied units is 5 while the number of contract rental units is 16. The total number of units is thus 21. For Tract 14, Block 105's Negroes, the number of owner-occupied units is 5 while the number of contract units is 16. So there are 21 units.

B. Divide the number of each type of unit by the total number of units on the block. This gives a weight to each.

Example: For both the total population of Block 105 and the Negro population, (which are identical in this block) 5 divided by 21 is .24, the owners weight, while 16 divided by 24 is .76, the renters weight.

C. Multiply the renter income figure by the renters weight and the owner-occupied by owner-occupied weight.

Example: For the total population of Block 105, multiplying the weights times the relevant income yields:

.24 times $5,698 = $1,367.52

.76 times $5,015 = $3,811.40

For Block 105's Negro population:

.24 times $5,668 = $1,360.32

.76 times $5,095 = $3,872.20

D. Add these two numbers together to obtain the total income figure.

Example: For the total population, $1,367.52 plus $3,811.40 equals $5,178.92, the family income number assigned to Block 105, Tract 14.

For the Negro population, $1,360.32 plus $3,872.20 gives $5,232.52, the Negro family income assigned to Block 105, Tract 14's Negro families.

SECTION 4

COMPUTER PACKAGE FOR BLOCK INCOME PROCEDURE

```
C
C       PURPOSE -- TO COMPUTE CUMULATIVE COUNTS AND CUMULATIVE
C       PERCENTAGES FOR TRACT INCOME DISTRIBUTIONS
C
C       SAME PROGRAM CAN BE USED FOR RENTER/OWNER DISTRIBUTIONS.
C
        DIMENSION NCTTOT(100),NCTBLK(100),NCTNBK(100)
        DIMENSION NCCTOT(100),NCCBLK(100),NCCNBK(100)
        DIMENSION CPTOT(100),CPBLK(100),CPNBK(100)
        NII=15
C
C       NII IS THE NUMBER OF INCOME INTERVALS
C
  1     READ (1) NTRACT
        READ (1) (NCTTOT(I),I=1,NII)
        READ (1) (NCTBLK(I),I=1,NII)
C
C       NTRACT IS THE TRACT NUMBER
C       NCTTOT(I) IS THE TOTAL COUNT FOR THE ITH INCOME INTERVAL
C       NCTBLK(I) IS THE BLACK COUNT FOR ITH INCOME INTERVAL
C
        NSMTOT=0
        NSMBLK=0
        NSMNBK=0
        DO 11 I=1,NII
           NCTNBK(I)=NCTTOT(I)-NCTBLK(I)
           NSMTOT=NSMTOT+NCTTOT(I)
           NSMBLK=NSMBLK+NCTBLK(I)
           NSMNBK=NSMNBK+NCTNBK(I)
           NCCTOT(I)=NSMTOT
           NCCBLK(I)=NSMBLK
  11       NCCNBK(I)=NSMNBK
C
C       NCTNBK(I) IS THE NON-BLACK COUNT FOR THE ITH INCOME INTERVAL
C       NSMTOT, NSMBLK, AND NSMNBK ARE SUMS OF THE RESPECTIVE COUNTS
C       NCCTOT, NCCBLK, AND NCCNBK ARE CUMULATIVE COUNTS OF THE RESPECTIVE
C          COUNTS.
C
        DO 21 I=1,NII
           CPTOT(I)=100.0*NCCTOT(I)/NSMTOT
           CPBLK(I)=100.0*NCCBLK(I)/NSMBLK
  21       CPNBK(I)=100.0*NCCNBK(I)/NSMNBK
C
C       CPTOT, CPBLK, AND CPNBK ARE CUMULATIVE PERCENTAGES FOR THE
C          RESPECTIVE COUNTS.
C
        DO 31 I=1,NII
  31       WRITE(3) I,NCTTOT(I),NCCTOT(I),CPTOT(I),NCTBLK(I),NCCBLK(I),
     1    CPBLK(I),NCTNBK(I),NCCNBK(I),CPNBK(I)
        GO TO 1
        END
```

```
++WRITE PRINT,YESCCCC2SD
C          DATA SET YESCC002SD AT LEVEL 002 AS OF 01/14/75                    CCCC01
      DIMENSION CDVAL(330),CCL(330,2),XIVAL(45),XOVAL(30),XRVAL(36),CCNTCCCC01
     X(3),OWN(3),RCNT(3),RNT(3),RO(3),RR(3),PERI(45),PERO(30),PERR(36),ECCCC02
      XSTY(3),AY(3)                                                          CCCC03
      DIMENSION CL(3,3),YO(3,3),A(3,3),ADJ(3,3,3),CD(7,3,3),PC(7,3,3),       CCC04
     1CUMI(45),CUMO(30),CUMR(36),PC(3)                                       CCC05
      DIMENSION Y(3),B(3),P(3)                                               CCC06
      DIMENSION ASTER(3)                                                     CCC07
      DEFINE FILE 7(15250,20,L,I7)                                           CCC08
      DATA I7,ASTER,BLANK,STAR/1,4*' ',' *'/                                 CCC09
      DATA PLUS/' +'/                                                        CCC10
      DATA SIGN/' -'/                                                        CCC11
      DATA PC/.25,.50,.75/                                                   CCC12
      CALL OPSYS('FILECPT',6,2000)                                           CCC13
C                                                                            CCC14
C   READ IN NORMAL CURVE AND DISTRIBUTION VALUES                             CCC15
C                                                                            CCC16
      READ(4) CDVAL,CCL,XIVAL,XOVAL,XRVAL                                    CCC17
      XOVAL(8 )=100000.                                                      CCC18
      XOVAL(19)=100000.                                                      CCC19
      XOVAL(30)=100000.                                                      CCC20
      DO 500 I=1,330                                                         CCC21
      DO 900 J=1,2                                                           CCC22
  900 CCL(I,J)=CCL(I,J)/100.                                                 CCC23
    8 FORMAT(3(15F8.0/)/8F8.0/2(11F8.0/)/8F8.0/2(14F8.0/))                   CCC24
    9 FORMAT(3F15.5)                                                         CCC25
      IT = 0                                                                 CCC26
C                                                                            CCC27
C   READ IN BLOCK DATA                                                       CCC28
C                                                                            CCC29
 1000 READ(5,END=9999) ITR,IBL,CCNT,OWN,RCNT,RNT                             CCC30
   10 FORMAT(2I5,3F10.0,3F10.0,3F10.0,3F10.2)                                CCC31
      IF(IT.LT.ITR) GO TO 1010                                               CCC32
      GO TO 1020                                                             CCC33
C                                                                            CCC34
C   READ IN TRACT DATA                                                       CCC35
C                                                                            CCC36
 1010 READ(6,11,IT,RO,RR,CL,YO,A,AY,CD,PC,ADJ,PERI,CUMI,PERO,CUMC,PERR,CUMRCCC37
     1,(P(J),Y(J),B(J),J=1,3)                                                CCC38
   11 FORMAT(2I5,2F10.2)                                                     CCC39
C                                                                            CCC40
C   LOOP ON RACE                                                             CCC41
C                                                                            CCC42
 1020 DO 5000 J=1,3                                                          CCC43
      IF(ITR.EQ.19.AND.J.NE.1)GO TO 1090                                     CCC44
      IF(RO(J).LE.0)RO(J)=RO(1)                                              CCC45
      IF(RR(J).LE.0)RR(J)=RR(1)                                              CCC46
C                                                                            CCC47
C   FIND BEGINNING AND END OF DISTRIBUTION FOR RACE J                        CCC48
C                                                                            CCC49
      NR=(J/2+1)*J**2+J-1                                                    CCC50
      NC=(J-1)*11+8                                                          CCC51
```

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      NM=J*J**2+J/2*(5-J**2)                                          CC052
      NK=(J-1)*14+8                                                   CC053
      NI=(J-1)*15+1                                                   CC054
      NY=J*15                                                         CC055
      IF(CUMI(15).GE.5C..AND.CUMI(NY).GE.2O.)GO TC 2CCC               CC056
 1090 ESTY(J)=0.                                                      CC057
      GO TO 50CC                                                      CC058
 2000 IF(CCNT(J).EQ.C..OR.OWN(J).EQ.0.) GO TO 2080                    CC059
C                                                                     CC060
C   FIND CWNER PERCENTILE                                             CC061
C                                                                     CC062
      DO 2010 I=NR,NC                                                 CC063
      IF(CWN(J).LT.XCVAL(I)) GC TC 2020                               CC064
 201C CONTINUE                                                        CC065
      PERTO = PERC(NC)                                                CC066
      GO TO 2030                                                      CC067
 2020 CALL NTRP(PERTC,XOVAL(I),XOVAL(I-1),OWN(J),PERC(I),PERC(I-1),I,NR)CC068
C                                                                     CC069
C   ADJUST OWNER PERCENTILE TCWARD MIDPOINT                           CC070
C                                                                     CC071
 2030 CALL NORM(PERTC,COL,CDVAL,FPER,RC(J))                           CC072
    3 FCRMAT(T20,'OWN',2F15.4)                                        CC073
C                                                                     CC074
C   FIND CWNER INCOME VALUE                                           CC075
C                                                                     CC076
      DO 2060 I=NI,NY                                                 CC077
      IF(FPER.LE.PERI(I)) GC TC 2070                                  CC078
 2060 CCNTINUE                                                        CC079
      GC TO 208C                                                      CC080
 207C IF(I.EQ.NY)GO TC 2075                                           CC081
      CALL NTRP(XNCO,PERI(I),PERI(I-1),FPER,XIVAL(I),XIVAL(I-1),I,NI) CC082
      GO TO 2076                                                      CC083
 2075 NERR=1                                                          CC084
      CALL FINDP(Y(J),XIVAL,PERI,FPER,J,APER)                         CC085
      IF(APER.EQ.C)WRITE(3,25)NERR,APER                               CC086
      NERR=2                                                          CC087
      IF(B(J).EQ.0)WRITE(3,25)NERR,B(J)                               CC088
      XNCO=Y(J)/APER**(1./B(J))                                       CC089
C                                                                     CC090
C   FIND CWNER ADJUSTMENT FACTOR                                      CC091
C                                                                     CC092
 2076 IF(FPER.LE..25)GO TO 2077                                       CC093
      IF(FPER.GE..75)GC TC 2078                                       CC094
      K=2                                                             CC095
      IF(FPER.GT..50)K=3                                              CC096
      CALL NTRP(ADJO,PC(K),PC(K-1),FPER,ADJ(J,1,K),ADJ(J,1,K-1),1,K)  CC097
      GC TO 2079                                                      CC098
 2077 ADJC=ADJ(J,1,2)                                                 CC099
      GC TO 2079                                                      CC100
 2078 ADJC=ADJ(J,1,3)                                                 CC101
C                                                                     CC102
C   FIND ADJUSTED OWNER INCCME                                        CC103
C                                                                     CC104
```

```
 2079 IF(ADJO.NE.0)XNCC=XNCC*ADJO                                       CC105
      GO TO 30CC                                                        CC106
 2080 XNCC = 0.                                                         CC107
 3000 IF(RCNT(J).EQ.0..OR.RNT(J).EQ.0.) GO TO 3080                      CC108
C                                                                       CC109
C   FIND RENT  PERCENTILE                                               CC110
C                                                                       CC111
      DO 3010 I=NM,NK                                                   CC112
      IF(RNT(J).LT.XRVAL(I)) GO TO 3020                                 CC113
 3010 CONTINUE                                                          CC114
      PERTR = PERR(NK)                                                  CC115
      GO TO 3030                                                        CC116
 3020 CALL NTRP(PERTR,XRVAL(I),XRVAL(I-1),RNT(J),PERR(I),PERR(I-1),I,NM)CC117
C                                                                       CC118
C   ADJUST RENT  PERCENTILE TOWARD MIDPOINT                             CC119
C                                                                       CC120
 3030 CALL NORM(PERTR,COL,CDVAL,FPER,RR(J))                             CC121
    4 FORMAT(T19,'RENT',2F15.4)                                         CC122
C                                                                       CC123
C   FIND RENT  INCOME VALUE                                             CC124
C                                                                       CC125
      DO 3060 I=NI,NY                                                   CC126
      IF(FPER.LE.PERI(I)) GO TO 3070                                    CC127
 3060 CONTINUE                                                          CC128
      GO TO 3080                                                        CC129
 3070 IF(I.EQ.NY)GO TO 3075                                             CC130
      CALL NTRP(XNCR,PERI(I),PERI(I-1),FPER,XIVAL(I),XIVAL(I-1),I,NI)   CC131
      GO TO 3076                                                        CC132
 3075 NERR=3                                                            CC133
      CALL FINDP(Y(J),XIVAL,PERI,FPER,J,APER)                           CC134
      IF(APER.EQ.0.)WRITE(3,25)NERR,APER                                CC135
      NERR=4                                                            CC136
      IF(B(J).EQ.0.0)WRITE(3,25)NERR,B(J)                               CC137
      XNCR=Y(J)/APER**(1./B(J))                                         CC138
C                                                                       CC139
C   FIND RENT  ADJUSTMENT FACTOR                                        CC140
C                                                                       CC141
 3076 IF(FPER.LE..25)GO TO 3077                                         CC142
      IF(FPER.GE..75)GO TO 3078                                         CC143
      K=2                                                               CC144
      IF(FPER.GT..50)K=3                                                CC145
      CALL NTRP(ADJR,PC(K),PC(K-1),FPER,ADJ(J,2,K),ADJ(J,2,K-1),1,K)    CC146
      GO TO 3079                                                        CC147
 3077 ADJR=ADJ(J,2,1)                                                   CC148
      GO TO 3079                                                        CC149
 3078 ADJR=ADJ(J,2,3)                                                   CC150
C                                                                       CC151
C   FIND ADJUSTED RENT  INCOME                                          CC152
C                                                                       CC153
 3079 IF(ADJR.NE.0)XNCR=XNCR*ADJR                                       CC154
      GO TO 4000                                                        CC155
 3080 XNCR = 0.                                                         CC156
C                                                                       CC157
```

```
C   FIND WEIGHTED AVERAGE OF OWNER AND RENT INCOMES                    CC158
C                                                                      CC159
 40CC IF(OCNT(J).EQ.C..OR.RCNT(J).EC.0.) GO TO 4010                    CC160
      NERR=5                                                           CC161
      IF((OCNT(J)+RCNT(J)).EQ.0.0) WRITE (3,25) NERR, OCNT(J), RCNT(J) CC162
   25 FORMAT ('0', 'ZERO AT CHECK  ',I1,3X,'VARIABLE EQUALS  ',2F15.5/) CC163
      ESTY(J) = XNCO * (OCNT(J) / (OCNT(J) + RCNT(J))) + XNCR * (RCNT(J)CC164
     X / (OCNT(J) + RCNT(J)))                                          CC165
      GO TO 5000                                                       CC166
 4010 ESTY(J) = XNCO + XNCR                                            CC167
    5 FORMAT(T17,'INCOME',2F15.4)                                      CC168
 5000 CONTINUE                                                         CC169
      IF(CUMI(30).LT.20)ESTY(3)=0                                      CC170
      DO 5005 J=1,3                                                    CC171
      IF(ITR.EQ.224.AND.(IBL.EQ.103.OR.IBL.EQ.104))GO TO 5004          CC172
      IF(ESTY(J).GT.C.)GOTO 5005                                       CC173
      IF(ESTY(1).GT.C..AND.ASTER(1).EQ.BLANK)GO TO 5003                CC174
 5002 ESTY(J)=AY(1)                                                    CC175
      ASTER(J)=SIGN                                                    CC176
      GO TO 5005                                                       CC177
 5003 ESTY(J)=ESTY(1)                                                  CC178
      ASTER(J)=PLUS                                                    CC179
      GO TO 5005                                                       CC180
 5004 ESTY(J)=AY(1)                                                    CC181
      ASTER(J)=STAR                                                    CC182
 5005 CONTINUE                                                         CC183
      WRITE(7'I7)ITR,IBL,ESTY                                          CC184
      WRITE(3,1)ITR,IBL,(ESTY(J),ASTER(J),J=1,3)                       CC185
      DO 5010 J=1,3                                                    CC186
 5010 ASTER(J)=BLANK                                                   CC187
      GO TO 1000                                                       CC188
    1 FORMAT(2I15,3(F20.4,A2))                                         CC189
    2 FORMAT(4F25.4)                                                   CC190
 9999 WRITE(3,12)I7                                                    CC191
   12 FORMAT(I10,' RECORDS WRITTEN TO DISK')                           CC192
      ITR=366                                                          CC193
      WRITE(7'I7)ITR                                                   CC194
      STOP                                                             CC195
    7 FORMAT(' RENT INCOME',2I5,F8.4,F12.2)                            CC196
    6 FORMAT('  OWN INCOME',2I5,F8.4,F12.2)                            CC197
      END                                                              CC198
      SUBROUTINE NORM(PER,COL,COVAL,FPER,R)                            CC199
      DIMENSION COL(330,2),COVAL(330)                                  CC200
      IF(PER.LT.C.50)GO TO 20                                          CC201
      L = 1                                                            CC202
      DO 10 I=1,330                                                    CC203
      IF(PER.LE.COL(I,L)) GO TO 50                                     CC204
   10 CONTINUE                                                         CC205
      GO TO 40                                                         CC206
   20 L = 2                                                            CC207
      DO 30 I=1,330                                                    CC208
      IF(PER.GE.COL(I,L)) GO TO 50                                     CC209
   30 CONTINUE                                                         CC210
```

```
   40 VALN=370.                                                          CC211
      GO TO 70                                                           CC212
   50 IF(PER.EQ.CCL(I,L)) GO TO 60                                       CC213
      CALL NTRP(VALN,CCL(I,L),CCL(I-1,L),PER,CDVAL(I),CDVAL(I-1),I,1)    CC214
      GO TO 70                                                           CC215
   60 VALN = CDVAL(I)                                                    CC216
   70 VALN = VALN * R                                                    CC217
      DO 80 I=1,330                                                      CC218
      IF(VALN.LE.CDVAL(I)) GO TO 90                                      CC219
   80 CONTINUE                                                           CC220
      WRITE(3,1)PER,R,VALN                                               CC221
    1 FORMAT('   BAD NORM,PER,R,VALN',3F14.8)                           CC222
      FPER=COL(330,L)                                                    CC223
      RETURN                                                             CC224
   90 IF(VALN.EQ.CDVAL(I)) GO TO 100                                     CC225
      CALL NTRP(FPER,CDVAL(I),CDVAL(I-1),VALN,CCL(I,L),COL(I-1,L),I,1)   CC226
      GO TO 110                                                          CC227
  100 FPER = CCL(I,L)                                                    00228
  110 RETURN                                                             CC229
      END                                                                CC230
      SUBROUTINE NTRP(X,HK,XLK,XMK,HU,XLU,I,N)                           CC231
      IF(I.EQ.N) GO TO 10                                                CC232
      IF((HK-XLK).EQ.0)GO TO 30                                          CC233
      X = HU - ((HU - XLU) * ((HK - XMK) / (HK - XLK)))                  CC234
      GO TO 20                                                           CC235
   10 IF(HK.EQ.0) GO TO 30                                               CC236
      X=HU-(HU*((HK-XMK)/HK))                                            CC237
   20 RETURN                                                             00238
   30 X=0                                                                00239
      RETURN                                                             CC240
      END                                                                CC241
      SUBROUTINE FINDP(Y,XIVAL,PERI,FPER,J,APER)                         00242
      DIMENSION PERI(45),XIVAL(15)                                       CC243
      DO 101 I=6,10                                                      CC244
      IF(XIVAL(I).EQ.Y)GO TO 102                                         00245
  101 CONTINUE                                                           CC246
      WRITE(3,1)                                                         CC247
      APER=0                                                             00248
      RETURN                                                             00249
  102 K=(J-1)*15+I                                                       CC250
      APER=(1.-FPER)/(1.-PERI(K))                                        00251
      RETURN                                                             00252
    1 FORMAT(' BAD YC')                                                  CC253
      END                                                                CC254
***** ABOVE ACTION SATISFACTORILY COMPLETED *****

**INSERT WORK
/*
//GO.SYSIN DD *
***** ABOVE ACTION SATISFACTORILY COMPLETED *****
```

```
C   THIS PROGRAM COMPUTES ESTIMATED CLOSING INCOME VALUES FOR EACH
C   TRACT, USING THE INCOME DISTRIBUTION WHICH WILL BE USED IN THE
C   FINAL INCOME PROGRAM AND THE PARETO ESTIMATION EQUATION
C
C   THE A, B, AND C VALUES WERE COMPUTED IN A SEPARATE REGRESSION RUN
C   A VALUES ARE FOR THE TOTAL SAMPLE, B FOR BLACKS, C FOR NON-BLACKS
C   A1 IS LOWER LIMIT OF LOWEST INCOME CLASS USED IN THE REGRESSION
C   A2 IS COEFFICIENT COMPUTED IN REGRESSION
C   A3 IS PERCENTAGE IN HIGHEST INCOME CLASS
C
      DIMENSION CLOSE(365,9)
      DIMENSION DATA (411)
      DATA CLOSE/3285*0.0/
      CALL OPSYS('FILEOPT',12,2000)
      CALL OPSYS('FILEOPT',14,2000)
  101 READ(1,1,END=201)IT,A1,A2,A3,B1,B2,B3
      CLOSE(IT,1)=A1/((A3/2.)**(1./A2))
      CLOSE(IT,2)=A1
      CLOSE(IT,3)=A2
      IF(B1.GT.0)CLOSE(IT,7)=B1/((B3/2.)**(1./B2))
      CLOSE(IT,8)=B1
      CLOSE(IT,9)=B2
      GO TO 101
  201 READ(1,1,END=301)IT,C1,C2,C3
      IF(C1.GT.0)CLOSE(IT,4)=C1/((C3/2.)**(1./C2))
      CLOSE(IT,5)=C1
      CLOSE(IT,6)=C2
PAGE  1
```

```
      GO TO 201
  301 DO 302 I=1,365
      IF(I.EQ.26.OR.I.EQ.43.OR.I.EQ.49.OR.I.EQ.52.OR.I.EQ.59.OR.I.EQ.63
     1.OR.I.EQ.124.OR.I.EQ.150.OR.I.EQ.328.OR.I.EQ.350) GO TO 302
C
C   COMBINES OTHER PREVIOUSLY COMPUTED DATA WITH CLOSING VALJES TO GET
C   INPUT FOR FINAL INCOME PROGRAM
C
      READ(12)ITT,DATA
      IF(ITT.NE.1)GO TO 302
      WRITE(3,2)I,(CLOSE(I,J),J=1,9)
      WRITE(14)ITT,DATA,(CLOSE(I,J),J=1,9)
  302 CONTINUE
    1 FORMAT(I3,2(2F6.0,F5.0))
    2 FORMAT((I5,3(F12.2,F10.0,F10.3)))
      STOP
      END
 END OF DATA
```

```
C
C    THIS PROGRAM COMPUTES ESTIMATED CLOSING INCOME VALUES USING THE
C    INCOME DISTRIBUTION WHICH WILL BE USED IN THE INCOME AND HOUSING
C    CORRELATION PROGRAM AND THE PARETO ESTIMATION EQUATION
C
      DIMENSION O(7,7,3),R(8,8,3),OS(7,3),RS(7,3),S(7,3,3),X(2,3),N(1),
     1CLOSE(3,3),YO(3,3),A(3,3)
      REAL LOG(3)
      REAL*8 LBL(2)
      COMMON/REGVAL/VARE,STDE,RSQDA,RSQDU,FTEST,DURBW,ALPHA,
     1BETA(20),SALPHA,SBETA(20),TALPHA,TBETA(20)
      DATA LBL/'PER CENT','LGYO-LGY'/
      EQUIVALENCE(S(1,1,1),OS(1,1)),(S(1,1,2),RS(1,1))
      CALL OPSYS('FILEOPT',14,1C50)
      LOG(1)=ALOG1C(1CCCO.)
      LOG(2)=ALOG1C(15CCO.)
      LOG(3)=ALOG1O(25CCO.)
      X(2,1)=C.
      X(2,2)=LOG(1)-LOG(2)
      X(2,3)=LOG(1)-LOG(3)
      N(1)=2
C
C    READ IN OWNER AND RENTER INCOME DISTRIBUTIONS
C
  1CC READ(14,1,END=999)II,(((O(I,J,K),I=1,7),J=1,7),K=1,2),
     1(((R(I,J,K),I=1,8),J=1,8),K=1,2)
      IF(O(1,1,1).LT.C.OR.O(1,1,2).LT.C)GO TO 101
      DO 1C2 I=1,7
```

```
      DO 102 J=1,7
  102 O(I,J,3)=O(I,J,1)-O(I,J,2)
      GO TO 103
  101 O(1,1,3)=-1
  103 IF(R(1,1,1).LT.0.OR.R(1,1,2).LT.0)GO TO 104
      DO 105 I=1,8
      DO 105 J=1,8
  105 R(I,J,3)=R(I,J,1)-R(I,J,2)
      GO TO 106
  104 R(1,1,3)=-1
C
C   CONVERT DISTRIBUTIONS TO CUMULATIVE DISTRIBUTIONS
C
  106 DO 107 K=1,3
      DO 108 J=1,7
      OS(J,K)=0.
  108 RS(J,K)=0.
      IF(O(1,1,K).LT.0)GO TO 110
      DO 109 I=1,7
      DO 109 J=1,7
  109 OS(I,K)=OS(I,K)+O(I,J,K)
  110 IF(R(1,1,K).LT.0)GO TO 112
      DO 111 J=1,8
      RS(1,K)=RS(1,K)+R(1,J,K)
      DO 111 I=2,3
  111 RS(I-1,K)=RS(I-1,K)+R(I,J,K)
  112 DO 119 I=1,7
  119 S(I,K,3)=S(I,K,1)+S(I,K,2)
```

```
C
C   COMPUTE PERCENT CUMULATIVE DISTRIBUTIONS AND VARIABLES NEEDED
C   FOR REGRESSION
C
      DO 113 L=1,3
      CLOSE(K,L)=0.
      YO(K,L)=0.
      A (K,L)=0.
      IF(S(7,K,L).EQ.0)GO TO 113
      DO 114 I=1,2
  114 S(7-I,K,L)=S(7-I,K,L)+S(8-I,K,L)
      DO 115 I=1,3
  115 X(1,I)=ALOG10(S(4+I,K,L)/S(5,K,L))
      CALL REGPRT(0)
      CALL REGCON(0)
      CALL REGRES(X,2,3,1,3,1.1,N)
      IF(RSQDU.NE.0)GO TO 116
      CLOSE(K,L)=37500
      IF(IT.EQ.5.OR.IT.EQ.22.OR.IT.EQ.126.OR.IT.EQ.146)GO TO 113
      IF(IT.NE.8)GO TO 117
      CLOSE(K,L)=35461.36
      YO(K,L)=10000
      A(K,L)=1.308
      GO TO 113
  117 IF(IT.NE.21)GO TO 118
      CLOSE(K,L)=47821.09
      YO(K,L)=6000
      A(K,L)=1.949
```

```
      GO TO 113
  118 CLOSE(K,L)=31106.71
      YO(K,L)=9000
      A(K,L)=1.005
      GO TO 113
C
C   COMPUTE ESTIMATED CLOSING INCOME VALUE USING RESULTS
C   FROM REGRESSION
C
  116 CLOSE(K,L)=10000./(.5*S(7,K,L)/S(5,K,L))**(1./BETA(1))
      YO(K,L)=10000
      A(K,L)=BETA(1)
      IF(IT.NE.229)GO TO 113
      CLOSE(K,L)=111334.69
      A(K,L)=.412
  113 CONTINUE
  107 CONTINUE
      WRITE(3,4)IT,CLOSE,YO,A
      WRITE(12)IT,CLOSE,YO,A
      GO TO 100
  999 STOP
    1 FORMAT(I4,98F8.0/4X,128F8.0)
    2 FORMAT(7F10.0)
    3 FORMAT(' BAD REGR',3I5)
    4 FORMAT(//I5,9F8.0,9F7.0/5X,9F6.3)
    5 FORMAT(3F10.5)
    6 FORMAT(2F12.6)
      END
```

```
DERAL RESERVE BANK OF PHILADELPHIA                          VER      01/17/75
NVALET LIBRARY FRB                                          7.1      10.32.41

WRITE PRINT,YESOC100SD
       DATA SET YESO0100SD AT LEVEL C03 AS OF 01/17/75
                                                                      00001
  THIS PROGRAM COMPUTES CORRELATIONS (R VALUES) BETWEEN INCOME AND    00002
  HOUSING VALUES AND CUMULATIVE DISTRIBUTIONS OF INCOME BY HOUSING    00003
  TYPE, BY RACE                                                       00004
                                                                      00005
     REAL INCO(7),OWN(7),INCR(8),RENT(7),OWNINC(7,7),OWNVAL(7,7),     00006
    1RNTINC(8,7),RNTVAL(8,7)                                          00007
     DIMENSION CLOSE(3),OT(7,7),ON(7,7),OW(7,7),RT(8,8),RN(8,8),RW(8,8)00008
     DIMENSION CUMDST(7,6)                                            00009
     DIMENSION XBAR(2),STD(2),RX(4),R(3),B(2),D(2),T(2),COR(6),       00010
    1X(5000,2),Y(10000),OLDR(6),AVGINC(3)                             00011
     DIMENSION ALPHA(6),CL(6),YO(6)                                   00012
     EQUIVALENCE(CLOSE(1),CL(1))                                      00013
     DATA OWN/3500.,7500.,12500.,17500.,22500.,30000.,60000./         00014
     DATA INCO/1500.,4000.,6000.,3500.,12500.,20000.,50000./          00015
     DATA RENT/25.,50.,70.,90.,125.,175.,350./                        00016
     DATA INCR/1000.,2500.,4000.,6000.,8500.,12500.,20000.,50000./    00017
     CALL OPSYS('FILEOPT',14,32767)                                   00018
                                                                      00019
  SET UP LIMITS OF DISTRIBUTION RANGES                                00020
                                                                      00021
     DO 201 I=1,7                                                     00022
     DO 201 J=1,7                                                     00023
     OWNINC(I,J)=INCO(I)                                              00024
201  OWNVAL(I,J)=OWN(J)                                               00025
     DO 202 I=1,8                                                     00026
     DO 202 J=1,7                                                     00027
     RNTINC(I,J)=INCR(I)                                              00028
202  RNTVAL(I,J)=RENT(J)                                              00029
                                                                      00030
  FOR EACH TRACT READ IN CLOSING INCOME VALUE, OWN AND RENT           00031
  DISTRIBUTIONS BY INCOME, AND AVERAGE INCOME, ALL BY RACE            00032
                                                                      00033
 19  READ(11,END=999)ITC,CL    ,YO,ALPHA                              00034
     READ(14,2)ITR,OT,ON                                              00035
     READ(14,3)ITRACT,RT,RN                                           00036
     IF(ITC.NE.ITR)GO TO 998                                          00037
     READ(12)ITRR,OLDR,AVGINC                                         00038
     IF(ITR.NE.ITRR )GO TO 996                                        00039
     DO 203 I=1,7                                                     00040
     DO 203 J=1,6                                                     00041
203  CUMDST(I,J)=0.                                                   00042
     IF(OT(1,1).LT.0)GO TO 25                                         00043
                                                                      00044
  COMPUTE R AND DISTRIBUTION FOR ALL OWNERS                           00045
                                                                      00046
     NOBS=0                                                           00047
     DO 301 I=1,7                                                     00048
     DO 301 J=1,7                                                     00049
     IF(OT(I,J).LE.0)GO TO 301                                        00050
     IF(I.NE.7)GO TO 303                                              00051
```

```
      IF(CLOSE(1).EQ.0)GO TO 304                                        00052
      OWNINC(I,J)=CLOSE(1)                                              00053
      GO T O 303                                                        00054
  304 OWNINC(I,J)=37500.                                                00055
      WRITE(3,106)ITC                                                   00056
  303 LIM1=NOBS+1                                                       00057
      NOBS=NOBS+OT(I,J)                                                 00058
      IF(NOBS.GT.5000)GO TO 997                                         00059
      DO 302 K=LIM1,NOBS                                                00060
      X(K,1)=OWNINC(I,J)                                                00061
  302 X(K,2)=OWNVAL(I,J)                                                00062
  301 CONTINUE                                                          00063
      DO 306 I=1,7                                                      00064
      DO 306 J=1,7                                                      00065
  306 CUMOST(I,1)=CUMDST(I,1)+OT(I,J)                                   00066
      CALL MOVE(X,Y,NOBS)                                               00067
      CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                        00068
      COR(1)=R(2)                                                       00069
    6 IF(ON(1,1).LT.0)GO TO 26                                          00070
                                                                        00071
    COMPUTE R AND DISTRIBUTION FOR BLACK OWNERS                         00072
                                                                        00073
                                                                        00074
      NOBS=0                                                            00075
      DO 311 I=1,7                                                      00076
      DO 311 J=1,7                                                      00077
      IF(ON(I,J).LE.0)GO TO 311                                         00078
      IF(I.NE.7)GO TO 313                                               00079
      IF(CLOSE(2).EQ.0)GO TO 314                                        00080
      OWNINC(I,J)=CLOSE(2)                                              00081
      GO T O 313                                                        00082
   14 OWNINC(I,J)=37500.                                                00083
      IF(CLOSE(1).GT.0)OWNINC(I,J)=CLOSE(1)                             00084
      WRITE(3,106)ITC                                                   00085
  313 LIM1=NOBS+1                                                       00086
      NOBS=NOBS+ON(I,J)                                                 00087
      IF(NOBS.GT.5000)GO TO 997                                         00088
      DO 312 K=LIM1,NOBS                                                00089
      X(K,1)=OWNINC(I,J)                                                00090
  312 X(K,2)=OWNVAL(I,J)                                                00091
  311 CONTINUE                                                          00092
      DO 316 I=1,7                                                      00093
      DO 316 J=1,7                                                      00094
  316 CUMOST(I,2)=CUMDST(I,2)+ON(I,J)                                   00095
      CALL MOVE(X,Y,NOBS)                                               00096
      CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                        00097
      COR(2)=R(2)                                                       00098
    0 IF(OT(1,1).LT.0.OR.ON(1,1).LT.0)GO TO 70                          00099
                                                                        00100
    COMPUTE R AND DISTRIBUTION FOR NON-BLACK OWNERS                     00101
                                                                        00102
      DO 401 I=1,7                                                      00103
      DO 401 J=1,7                                                      00104
  401 OW(I,J)=OT(I,J)-ON(I,J)
```

```
      NOBS=0                                                          00105
      DO 321 I=1,7                                                    00106
      DO 321 J=1,7                                                    00107
      IF(OW(I,J).LE.C)GO TO 321                                       00108
      IF(I.NE.7)GO TO 323                                             00109
      IF(CLOSE(3).EQ.0)GO TO 324                                      00110
      OWNINC(I,J)=CLOSE(3)                                            00111
      GO T O 323                                                      00112
 24   OWNINC(I,J)=37500.                                              00113
      IF(CLOSE(1).GT.0)OWNINC(I,J)=CLOSE(I)                           00114
      WRITE(3,106)ITC                                                 00115
 323  LIM1=NOBS+1                                                     00116
      NOBS=NOBS+OW(I,J)                                               00117
      IF(NOBS.GT.5000)GO TO 997                                       00118
      DO 322 K=LIM1,NOBS                                              00119
      X(K,1)=OWNINC(I,J)                                              00120
 322  X(K,2)=OWNVAL(I,J)                                              00121
 321  CONTINUE                                                        00122
      IF(NOBS.EQ.0)GO TO 70                                           00123
      DO 326 I=1,7                                                    00124
      DO 326 J=1,7                                                    00125
 326  CUMDST(I,3)=CUMDST(I,3)+OW(I,J)                                 00126
      CALL MOVE(X,Y,NOBS)                                             00127
      CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                      00128
      COR(3)=R(2)                                                     00129
  75  IF(RT(1,1).LT.C)GO TO 35                                        00130
                                                                      00131
    COMPUTE R AND DISTRIBUTION FOR ALL RENTERS                        00132
                                                                      00133
      NOBS=0                                                          00134
      DO 331 I=1,8                                                    00135
      DO 331 J=1,7                                                    00136
      IF(RT(I,J).LE.0)GO TO 331                                       00137
      IF(I.NE.8)GO TO 333                                             00138
      IF(CLOSE(4).EQ.0)GO TO 334                                      00139
      RNTINC(I,J)=CLOSE(4)                                            00140
      GO T O 333                                                      00141
 334  RNTINC(I,J)=37500.                                              00142
      WRITE(3,106)ITC                                                 00143
 333  LIM1=NOBS+1                                                     00144
      NOBS=NOBS+RT(I,J)                                               00145
      IF(NOBS.GT.5000)GO TO 997                                       00146
      DO 332 K=LIM1,NOBS                                              00147
      X(K,1)=RNTINC(I,J)                                              00148
 332  X(K,2)=RNTVAL(I,J)                                              00149
 331  CONTINUE                                                        00150
      DO 336 J=1,8                                                    00151
      CUMDST(1,4)=CUMDST(1,4)+RT(1,J)                                 00152
      DO 336 I=2,8                                                    00153
 336  CUMDST(I-1,4)=CUMDST(I-1,4)+RT(I,J)                             00154
      CALL MOVE(X,Y,NOBS)                                             00155
      CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                      00156
      COR(4)=R(2)                                                     00157
```

```
      IF(RN(1,1).LT.0)GO TO 36                                          00158
                                                                        00159
    COMPUTE R AND DISTRIBUTION FOR BLACK RENTERS                        00160
                                                                        00161
      NOBS=0                                                            00162
      DO 341 I=1,8                                                      00163
      DO 341 J=1,7                                                      00164
      IF(RN(I,J).LE.0)GO TO 341                                         00165
      IF(I.NE.8)GO TO 343                                               00166
      IF(CLOSE(5).EQ.0)GO TO 344                                        00167
      RNTINC(I,J)=CLOSE(5)                                              00168
      GO T O 343                                                        00169
  344 RNTINC(I,J)=37500.                                                00170
      IF(CLOSE(4).GT.0)RNTINC(I,J)=CLOSE(4)                             00171
      WRITE(3,106)ITC                                                   00172
  343 LIM1=NOBS+1                                                       00173
      NOBS=NOBS+RN(I,J)                                                 00174
      IF(NOBS.GT.5000)GO TO 997                                         00175
      DO 342 K=LIM1,NOBS                                                00176
      X(K,1)=RNTINC(I,J)                                                00177
  342 X(K,2)=RNTVAL(I,J)                                                00178
  341 CONTINUE                                                          00179
      DO 346 J=1,8                                                      00180
      CUMDST(1,5)=CUMDST(1,5)+RN(1,J)                                   00181
      DO 346 I=2,8                                                      00182
  346 CUMDST(I-1,5)=CUMDST(I-1,5)+RN(I,J)                               00183
      CALL MOVE(X,Y,NOBS)                                               00184
      CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                        00185
      COR(5)=R(2)                                                       00186
      IF(RN(1,1).LT.0.OR.RT(I,1).LT.0)GO TO 80                          00187
                                                                        00188
    COMPUTE R AND DISTRIBUTION FOR NON-BLACK RENTERS                    00189
                                                                        00190
      DO 402 I=1,8                                                      00191
      DO 402 J=1,8                                                      00192
  402 RW(I,J)=RT(I,J)-RN(I,J)                                           00193
      NOBS=0                                                            00194
      DO 351 I=1,8                                                      00195
      DO 351 J=1,7                                                      00196
      IF(RW(I,J).LE.0)GO TO 351                                         00197
      IF(I.NE.8)GO TO 353                                               00198
      IF(CLOSE(6).EQ.0)GO TO 354                                        00199
      RNTINC(I,J)=CLOSE(6)                                              00200
      GO T O 353                                                        00201
  354 RNTINC(I,J)=37500.                                                00202
      IF(CLOSE(4).GT.0)RNTINC(I,J)=CLOSE(4)                             00203
      WRITE(3,106)ITC                                                   00204
  353 LIM1=NOBS+1                                                       00205
      NOBS=NOBS+RW(I,J)                                                 00206
      IF(NOBS.GT.5000)GO TO 997                                         00207
      DO 352 K=LIM1,NOBS                                                00208
      X(K,1)=RNTINC(I,J)                                                00209
  352 X(K,2)=RNTVAL(I,J)                                                00210
```

```
 1 CONTINUE                                                              00211
   IF(NOBS.EQ.0)GO TO 80                                                 00212
   DO 356 J=1,8                                                          00213
   CUMDST(1,6)=CUMDST(1,6)+RW(1,J)                                       00214
   DO 356 I=2,8                                                          00215
 6 CUMDST(I-1,6)=CUMDST(I-1,6)+RW(I,J)                                   00216
   CALL MOVE(X,Y,NOBS)                                                   00217
   CALL CORRE(NOBS,2,1,Y,XBAR,STD,RX,R,B,D,T)                            00218
   COR(6)=R(2)                                                           00219
 35 IF(ITR.EQ.14)WRITE(3,9)CUMDST                                        00220
                                                                         00221
 COMPUTE DISTRIBUTIONS FOR OWNERS AND RENTERS COMBINED                   00222
                                                                         00223
   DO 204 J=1,6                                                          00224
   DO 204 I=2,7                                                          00225
 04 CUMDST(I,J)=CUMDST(I,J)+CUMDST(I-1,J)                                00226
   IF (ITR.EQ.14)WRITE(3,9)CUMDST                                        00227
   WRITE(3,5)ITR,COR                                                     00228
   WRITE(13)ITR,COR,CL,AVGINC,YO,ALPHA,CUMDST                            00229
   GO TO 19                                                              00230
   WRITE(3,101)ITRACT                                                    00231
   COR(1)=0                                                              00232
   GO TO 16                                                              00233
   WRITE(3,102)ITRACT                                                    00234
   COR(2)=0                                                              00235
   GO TO 30                                                              00236
   WRITE(3,103)ITRACT                                                    00237
   COR(4)=0                                                              00238
   GO TO 40                                                              00239
   WRITE(3,104)ITRACT                                                    00240
   COR(5)=0                                                              00241
   GO TO 50                                                              00242
 70 COR(3)=0                                                             00243
   GO TO 75                                                              00244
 80 COR(6)=0                                                             00245
   GO TO 85                                                              00246
 96 WRITE(3,3)ITR,ITRR                                                   00247
   STOP                                                                  00248
 97 WRITE(3,4)                                                           00249
   STOP                                                                  00250
 98 WRITE(3,1)ITC,ITR                                                    00251
 99 STOP                                                                 00252
  1 FORMAT(' TRACTS DONT MATCH',2I5)                                     00253
    FORMAT(  I4,98F8.0)                                                  00254
  3 FORMAT(I4,128F8.0)                                                   00255
  9 FORMAT(7F8.0)                                                        00256
  4 FORMAT(' TOO MANY OBSERVATIONS')                                     00257
    FORMAT('0','TRACT=',I3,' TOT OWN = ',F7.4,' NEG OWN= ',F7.4,' WHIT00258
    FORMAT('0','TRACT=',I3,' TOT OWN = ',F7.4,' NEG OWN= ',F7.4,' WHITE  00259
   1E OWN = ',F7.4/9X,'TOT RENT =',F7.4,' NEGRO RENT =',F7.4,' WHITE     00260
   2RENT = ',F7.4)                                                       00261
  6 FORMAT(2F10.0)                                                       00262
  7 FORMAT(F10.0)                                                        00263
  8 FORMAT(' AVGINC TRACT DOESNT MATCH',2I5)
```

```
  101 FORMAT('0','   OWN TOTAL SUPPRESSED, TRACT = ',I3)                          00264
  102 FORMAT('0','   OWN NEG   SUPPRESSED, TRACT = ',I3)                          00265
  103 FORMAT('0','   RENT TOTAL SUPPRESSED, TRACT = ',I3)                         00266
  104 FORMAT('0','   RENT NEG   SUPPRESSED, TRACT = ',I3)                         00267
  105 FORMAT('0',' FOR TRACT ',I3,' NO OF TOT OWN ',I6,' NO OF NEG OWN',00268
     1I6,' NO OF TOT RENT ',I6,' NO OF NEG RENT ',I6,' NO OWN SUBTRACTED00269
     2 ',I6,/' NO OF RENT SUBTRACTED ',I6)                                        00270
  106 FORMAT(' DATA IN HIGHEST CLASS WITH NO CLOSING VALUE',I5)                   0027
      END                                                                         00272
      SUBROUTINE MOVET(X,Y,N)                                                     00273
                                                                                  00274
    CHANGE FORMAT OF ARRAY FOR CALL TO CORRE                                      00275
                                                                                  00276
      DIMENSION X(5000,2),Y(10000)                                                00277
      DO 101 I=1,N                                                                00278
      Y(I)=X(I,1)                                                                 00279
  101 Y(N+I)=X(I,2)                                                               00280
      RETURN                                                                      00281
      END                                                                         00282
      SUBROUTINE CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)                             00283
      DIMENSION X(1),XBAR(1),STD(1),RX(1),R(1),B(1),D(1),T(1)                     00284
      DO 100 J=1,M                                                                00285
      B(J)=0.0                                                                    00286
  100 T(J)=0.0                                                                    00287
      K=(M*M+M)/2                                                                 00288
      DO 102 I=1,K                                                                00289
  102 R(I)=0.0                                                                    00290
      FN=N                                                                        00291
      L=0                                                                         00292
      IF(IO) 105, 127, 105                                                        00293
  105 DO 108 J=1,M                                                                00294
      DO 107 I=1,N                                                                00295
      L=L+1                                                                       00296
  107 T(J)=T(J)+X(L)                                                              00297
      XBAR(J)=T(J)                                                                00298
  108 T(J)=T(J)/FN                                                                00299
      DO 115 I=1,N                                                                00300
      JK=0                                                                        00301
      L=I-N                                                                       00302
      DO 110 J=1,M                                                                00303
      L=L+N                                                                       00304
      D(J)=X(L)-T(J)                                                              00305
  110 B(J)=B(J)+D(J)                                                              00306
      DO 115 J=1,M                                                                00307
      DO 115 K=1,J                                                                00308
      JK=JK+1                                                                     00309
  115 R(JK)=R(JK)+D(J)*D(K)                                                       00310
      GO TO 205                                                                   00311
  127 IF(N-M) 130, 130, 135                                                       00312
  130 KK=N                                                                        00313
      GO TO 137                                                                   00314
  135 KK=M                                                                        00315
  137 DO 140 I=1,KK                                                               00316
```

```
      CALL DATA (M,D)                                                   00317
      DO 140 J=1,M                                                      00318
      T(J)=T(J)+D(J)                                                    00319
      L=L+1                                                             00320
  140 RX(L)=D(J)                                                        00321
      FKK=KK                                                            00322
      DO 150 J=1,M                                                      00323
      XBAR(J)=T(J)                                                      00324
  150 T(J)=T(J)/FKK                                                     00325
      L=0                                                               00326
      DO 180 I=1,KK                                                     00327
      JK=0                                                              00328
      DO 170 J=1,M                                                      00329
      L=L+1                                                             00330
  170 D(J)=RX(L)-T(J)                                                   00331
      DO 180 J=1,M                                                      00332
      B(J)=B(J)+D(J)                                                    00333
      DO 180 K=1,J                                                      00334
      JK=JK+1                                                           00335
  180 R(JK)=R(JK)+D(J)*D(K)                                             00336
      IF(N-KK) 205, 205, 185                                            00337
  185 KK=N-KK                                                           00338
      DO 200 I=1,KK                                                     00339
      JK=0                                                              00340
      CALL DATA (M,D)                                                   00341
      DO 190 J=1,M                                                      00342
      XBAR(J)=XBAR(J)+D(J)                                              00343
      D(J)=D(J)-T(J)                                                    00344
  190 B(J)=B(J)+D(J)                                                    00345
      DO 200 J=1,M                                                      00346
      DO 200 K=1,J                                                      00347
      JK=JK+1                                                           00348
  200 R(JK)=R(JK)+D(J)*D(K)                                             00349
  205 JK=0                                                              00350
      DO 210 J=1,M                                                      00351
      XBAR(J)=XBAR(J)/FN                                                00352
      DO 210 K=1,J                                                      00353
      JK=JK+1                                                           00354
  210 R(JK)=R(JK)-B(J)*B(K)/FN                                          00355
      JK=0                                                              00356
      DO 220 J=1,M                                                      00357
      JK=JK+J                                                           00358
  220 STD(J)= SQRT( ABS(R(JK)))                                         00359
      DO 230 J=1,M                                                      00360
      DO 230 K=J,M                                                      00361
      JK=J+(K*K-K)/2                                                    00362
      L=M*(J-1)+K                                                       00363
      RX(L)=R(JK)                                                       00364
      L=M*(K-1)+J                                                       00365
      RX(L)=R(JK)                                                       00366
      IF(STD(J)*STD(K)) 225, 222, 225                                   00367
  222 R(JK)=0.0                                                         00368
      GO TO 230                                                         00369
```

```
 225 R(JK)=R(JK)/(STD(J)*STD(K))                                  00370
 230 CONTINUE                                                     00371
     FN=SQRT(FN-1.0)                                              00372
     DO 240 J=1,M                                                 00373
 240 STD(J)=STD(J)/FN                                             00374
     L=-M                                                         00375
     DO 250 I=1,M                                                 00376
     L=L+M+1                                                      00377
 250 B(I)=RX(L)                                                   00378
     RETURN                                                       00379
     END                                                          00380
**** ABOVE ACTION SATISFACTORILY COMPLETED *****
```

```
C
C   THIS PROGRAM COMPUTES ADJUSTMENT FACTORS FOR OWNERS AND RENTERS
C   AT THE 25TH, 50TH, AND 75TH PERCENTILES, BY RACE, TO BE USED
C   IN THE FINAL INCOME PROGRAM.   THE ADJUSTMENT FACTOR IS THE
C   RATIO OF THE OWNER OR RENTER INCOME AT EACH PERCENTILE TO THE
C   COMBINED OWNER AND RENTER INCOME AT THAT PERCENTILE.
C
      DIMENSION R(6),CL(9),AVG(3),YO(9),A(9),CD(7,9),PD(7,9),ADJ(9,3),
     1X(7),FILL(21)
      DATA X/3000.,5000.,7000.,10000.,15000.,25000.,50000.    /
      EQUIVALENCE (FILL(7),AVG(1))
  100 READ(14,END=999)IT,R,FILL,CD,PD
      READ(11)ITT,CL,YO,A
C
C   COMPUTE INCOME AT EACH PERCENTILE BY HOUSING TYPE AND BY RACE
C
      DO 111 J=1,9
      IF(PD(7,J).EQ.0)GO TO 107
      DO 101 N=1,3
      PC=.25*N
      DO 102 I=1,7
      IF(PC.EQ.PD(I,J))GO TO 103
      IF(PC.LT.PD(I,J))GO TO 104
  102 CONTINUE
      WRITE(3,1)IT,PC,J,PD(7,J)
      ADJ(J,N)=0.
      GO TO 101
  104 IF(I.EQ.1)GO TO 105
```

```
      IF(I.LT.7)GO TO 112
      IF(YO(J).GT.0)GO TO 113
      WRITE(3,5)IT,J,PC
      GO TO 112
113   ADJ(J,N)=YO(J)/(1.-PC)**(1./A(J))
      IF(IT.EQ.21.AND.J.EQ.3.AND.N.EQ.3)ADJ(3,3)=ADJ(3,2)
      IF(IT.EQ.208.AND.J.EQ.1.AND.N.EQ.3)ADJ(1,3)=ADJ(1,2)
      IF(IT.EQ.226.AND.J.EQ.1.AND.(N.EQ.1.OR.N.EQ.3))ADJ(1,N)=88271.94
      WRITE(3,4)IT,J,PC,ADJ(J,N)
112   ADJ(J,N)=X(I-1)+((X(I)-X(I-1))*(PC-PD(I-1,J))/(PD(I,J)-PD(I-1,J)))
      GO TO 101
105   ADJ(J,N)=X(I)*PC/PD(I,J)
      GO TO 101
103   ADJ(J,N)=X(I)
101   CONTINUE
      GO TO 111
107   DO 108 N=1,3
108   ADJ(J,N)=0
111   CONTINUE
      IF(IT.EQ.14)WRITE(3,2)ADJ

C     COMPUTE ADJUSTMENT AS RATIO OF INCOMES

      DO 106 I=1,6
      DO 106 J=1,3
      L=MOD(I-1,3)+7
      IF(ADJ(L,J).EQ.0)GO TO 109
      ADJ(I,J)=ADJ(I,J)/ADJ(L,J)
GO    2
```

```
      GO TO 106
  109 ADJ(I,J)=0
  106 CONTINUE
      IF(IT.EQ.14)WRITE(3,3)ADJ
      WRITE(12)IT,R,CL,AVG,YO,A,CO,PD,ADJ
      GO TO 100
  999 STOP
    1 FORMAT(' BAD PC DIST',I5,F10.4,I5,F10.4)
    2 FORMAT(3F12.2)
    3 FORMAT(3F10.6)
    4 FORMAT(2I5,F5.2,F12.2)
    5 FORMAT(2I5,F5.2,'   WARNING')
      END
 END OF DATA
```

NORMAL DEVIATE TABLE

COLUMN 1 -- NORMAL DEVIATE VALUES
COLUMN 2 -- CUMULATIVE PERCENT .5 AND ABOVE
COLUMN 3 -- 1 MINUS CUMULATIVE PERCENT FOR OBSERVATIONS BELOW .5

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT. .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT. .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT. .5-
0.00	.5000	.5000	0.22	.5871	.4129	0.44	.6700	.3300
0.01	.5040	.4960	0.23	.5910	.4090	0.45	.6736	.3264
0.02	.5080	.4920	0.24	.5948	.4052	0.46	.6772	.3228
0.03	.5120	.4880	0.25	.5987	.4013	0.47	.6808	.3192
0.04	.5160	.4840	0.26	.6026	.3974	0.48	.6844	.3156
0.05	.5199	.4801	0.27	.6064	.3936	0.49	.6879	.3121
0.06	.5239	.4761	0.28	.6103	.3897	0.50	.6915	.3085
0.07	.5279	.4721	0.29	.6141	.3859	0.51	.6950	.3050
0.08	.5319	.4681	0.30	.6179	.3821	0.52	.6985	.3015
0.09	.5359	.4641	0.31	.6217	.3783	0.53	.7019	.2981
0.10	.5398	.4602	0.32	.6255	.3745	0.54	.7054	.2946
0.11	.5438	.4562	0.33	.6293	.3707	0.55	.7088	.2912
0.12	.5478	.4522	0.34	.6331	.3669	0.56	.7123	.2877
0.13	.5517	.4483	0.35	.6368	.3632	0.57	.7157	.2843
0.14	.5557	.4443	0.36	.6406	.3594	0.58	.7190	.2810
0.15	.5596	.4404	0.37	.6443	.3557	0.59	.7224	.2776
0.16	.5636	.4364	0.38	.6480	.3520	0.60	.7257	.2743
0.17	.5675	.4325	0.39	.6517	.3483	0.61	.7291	.2709
0.18	.5714	.4286	0.40	.6554	.3446	0.62	.7324	.2676
0.19	.5753	.4247	0.41	.6591	.3409	0.63	.7357	.2643
0.20	.5793	.4207	0.42	.6628	.3372	0.64	.7389	.2611
0.21	.5832	.4168	0.43	.6664	.3336	0.65	.7422	.2578

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-
0.66	.7454	.2546	0.88	.8106	.1894	1.10	.8643	.1357
0.67	.7486	.2514	0.89	.8133	.1867	1.11	.8665	.1335
0.68	.7517	.2483	0.90	.8159	.1841	1.12	.8686	.1314
0.69	.7549	.2451	0.91	.8186	.1814	1.13	.8708	.1292
0.70	.7580	.2420	0.92	.8212	.1788	1.14	.8729	.1271
0.71	.7611	.2389	0.93	.8238	.1762	1.15	.8749	.1251
0.72	.7642	.2358	0.94	.8264	.1736	1.16	.8770	.1230
0.73	.7673	.2327	0.95	.8289	.1711	1.17	.8790	.1210
0.74	.7704	.2296	0.96	.8315	.1685	1.18	.8810	.1190
0.75	.7734	.2266	0.97	.8340	.1660	1.19	.8830	.1170
0.76	.7764	.2236	0.98	.8365	.1635	1.20	.8849	.1151
0.77	.7794	.2206	0.99	.8389	.1611	1.21	.8869	.1131
0.78	.7823	.2177	1.00	.8413	.1587	1.22	.8888	.1112
0.79	.7852	.2148	1.01	.8438	.1562	1.23	.8907	.1093
0.80	.7881	.2119	1.02	.8461	.1539	1.24	.8925	.1075
0.81	.7910	.2090	1.03	.8485	.1515	1.25	.8944	.1056
0.82	.7939	.2061	1.04	.8508	.1492	1.26	.8962	.1038
0.83	.7967	.2033	1.05	.8531	.1469	1.27	.8980	.1020
0.84	.7995	.2005	1.06	.8554	.1446	1.28	.8997	.1003
0.85	.8023	.1977	1.07	.8577	.1423	1.29	.9015	.0985
0.86	.8051	.1949	1.08	.8599	.1401	1.30	.9032	.0968
0.87	.8078	.1922	1.09	.8621	.1379	1.31	.9049	.0951

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-
1.32	.9066	.0934	1.54	.9382	.0618	1.76	.9608	.0392
1.33	.9082	.0918	1.55	.9394	.0606	1.77	.9616	.0384
1.34	.9099	.0901	1.56	.9406	.0594	1.78	.9625	.0375
1.35	.9115	.0885	1.57	.9418	.0582	1.79	.9633	.0367
1.36	.9131	.0869	1.58	.9429	.0571	1.80	.9641	.0359
1.37	.9147	.0853	1.59	.9441	.0559	1.81	.9649	.0351
1.38	.9162	.0838	1.60	.9452	.0548	1.82	.9656	.0344
1.39	.9177	.0823	1.61	.9463	.0537	1.83	.9664	.0336
1.40	.9192	.0808	1.62	.9474	.0526	1.84	.9671	.0329
1.41	.9207	.0793	1.63	.9484	.0516	1.85	.9678	.0322
1.42	.9222	.0778	1.64	.9495	.0505	1.86	.9686	.0314
1.43	.9236	.0764	1.65	.9505	.0495	1.87	.9693	.0307
1.44	.9251	.0749	1.66	.9515	.0485	1.88	.9699	.0301
1.45	.9265	.0735	1.67	.9525	.0475	1.89	.9706	.0294
1.46	.9279	.0721	1.68	.9535	.0465	1.90	.9713	.0287
1.47	.9292	.0708	1.69	.9545	.0455	1.91	.9719	.0281
1.48	.9306	.0694	1.70	.9554	.0446	1.92	.9726	.0274
1.49	.9319	.0681	1.71	.9564	.0436	1.93	.9732	.0268
1.50	.9332	.0668	1.72	.9573	.0427	1.94	.9738	.0262
1.51	.9345	.0655	1.73	.9582	.0418	1.95	.9744	.0256
1.52	.9357	.0643	1.74	.9591	.0409	1.96	.9750	.0250
1.53	.9370	.0630	1.75	.9599	.0401	1.97	.9756	.0244

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-
1.98	.9761	.0239	2.20	.9861	.0139	2.42	.9922	.0078
1.99	.9767	.0233	2.21	.9864	.0136	2.43	.9925	.0075
2.00	.9772	.0228	2.22	.9868	.0132	2.44	.9927	.0073
2.01	.9778	.0222	2.23	.9871	.0129	2.45	.9929	.0071
2.02	.9783	.0217	2.24	.9875	.0125	2.46	.9931	.0069
2.03	.9788	.0212	2.25	.9878	.0122	2.47	.9932	.0068
2.04	.9793	.0207	2.26	.9881	.0119	2.48	.9934	.0066
2.05	.9798	.0202	2.27	.9884	.0116	2.49	.9936	.0064
2.06	.9803	.0197	2.28	.9887	.0113	2.50	.9938	.0062
2.07	.9808	.0192	2.29	.9890	.0110	2.51	.9940	.0060
2.08	.9812	.0188	2.30	.9893	.0107	2.52	.9941	.0059
2.09	.9817	.0183	2.31	.9896	.0104	2.53	.9943	.0057
2.10	.9821	.0179	2.32	.9898	.0102	2.54	.9945	.0055
2.11	.9826	.0174	2.33	.9901	.0099	2.55	.9946	.0054
2.12	.9830	.0170	2.34	.9904	.0096	2.56	.9948	.0052
2.13	.9834	.0166	2.35	.9906	.0094	2.57	.9949	.0051
2.14	.9838	.0162	2.36	.9909	.0091	2.58	.9951	.0049
2.15	.9842	.0158	2.37	.9911	.0089	2.59	.9952	.0048
2.16	.9846	.0154	2.38	.9913	.0087	2.60	.9953	.0047
2.17	.9850	.0150	2.39	.9916	.0084	2.61	.9955	.0045
2.18	.9854	.0146	2.40	.9918	.0082	2.62	.9956	.0044
2.19	.9857	.0143	2.41	.9920	.0080	2.63	.9957	.0043

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-
2.64	.9955	.0041	2.86	.9979	.0021	3.08	.9990	.0010
2.65	.9960	.0040	2.87	.9979	.0021	3.09	.9990	.0010
2.66	.9961	.0039	2.88	.9980	.0020	3.10	.9990	.0010
2.67	.9962	.0038	2.89	.9981	.0019	3.11	.9991	.0009
2.68	.9963	.0037	2.90	.9981	.0019	3.12	.9991	.0009
2.69	.9964	.0036	2.91	.9982	.0018	3.13	.9991	.0009
2.70	.9965	.0035	2.92	.9982	.0018	3.14	.9992	.0008
2.71	.9966	.0034	2.93	.9983	.0017	3.15	.9992	.0008
2.72	.9967	.0033	2.94	.9984	.0016	3.16	.9992	.0008
2.73	.9968	.0032	2.95	.9984	.0016	3.17	.9992	.0008
2.74	.9969	.0031	2.96	.9985	.0015	3.18	.9993	.0007
2.75	.9970	.0030	2.97	.9985	.0015	3.19	.9993	.0007
2.76	.9971	.0029	2.98	.9986	.0014	3.20	.9993	.0007
2.77	.9972	.0028	2.99	.9986	.0014	3.21	.9993	.0007
2.78	.9973	.0027	3.00	.9987	.0013	3.22	.9994	.0006
2.79	.9974	.0026	3.01	.9987	.0013	3.23	.9994	.0006
2.80	.9974	.0026	3.02	.9987	.0013	3.24	.9994	.0006
2.81	.9975	.0025	3.03	.9988	.0012	3.30	.9995	.0005
2.82	.9976	.0024	3.04	.9988	.0012	3.40	.9997	.0003
2.83	.9977	.0023	3.05	.9989	.0011	3.50	.9998	.0002
2.84	.9977	.0023	3.06	.9989	.0011	3.60	.9998	.0002
2.85	.9978	.0022	3.07	.9989	.0011	3.70	.9999	.0001

NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-	NORMAL DEV. VAL.	CUM. PCT. .5+	CUM. PCT.' .5-